# The Five-Number Summary <br> Lecture 16 Sections 5.3.1-5.3.3 

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## Outline

(1) Homework Review
(2) Percentiles and Quartiles

- Example
(3) The Five-Number Summary
- Examples

4 TI-83 Five-Number Summary
(5) The Interquartile Range

6 Percentiles in Excel
(7) Assignment

## Exercise 5.7, p. 312.

(a) The average (or mean) age for 10 adults in a room is 35 years. A 32-year-old adult new enters the room. Can you find the new average age for the 11 adults? If so, find it. If not, explain why not.
(b) The median age for 10 adults in a room is 35 years. A 32 -year-old adult new enters the room. Can you find the new median age for the 11 adults? If so, find it. If not, explain why not.

## Solution

(a) If the average age of 10 adults is 35 , then the total of their ages must be 350. The 32-year-old makes the total 382 , so the new average is

$$
\frac{382}{11}=34.73
$$

## Solution

(b) - In this case, we cannot find the new median. We know that half the people in the room are 35 or less, but we do not know how their ages are distributed.

- For example, if they are all 30, then the 32-year-old would be the new median.
- On the other hand, if they were all 34, then the new median would be 34.


## Percentiles and Quartiles

## Definition ( $p^{\text {th }}$ percentile)

The $p^{\text {th }}$ percentile of a set of numbers is a number that divides the lower $p \%$ of the numbers from the rest.

## Definition (1st quartile)

The 1 st quartile, denoted $Q_{1}$, of a set of numbers is the $25^{\text {th }}$ percentile.

## Definition (3rd quartile)

The 3rd quartile, denoted $Q_{3}$, of a set of numbers is the $75^{\text {th }}$ percentile.

## Finding Quartiles

Homework
Review
Percentiles and Quartiles

Example
The
Five-Number
Summary
Examples
Ti-83
Five-Number Summary

The
Interquartile
Range

- To find the quartiles, first find the position of the median.
- Then the 1 st quartile is the median of all the numbers that are below that position.
- The 3rd quartile is the median of all the numbers that are above that position.


## Example

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles
Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

## Example (Quartiles)

- Find the median and quartiles of the following sample.

$$
5,8,10,15,17,19,20,24,25,30,32
$$

## Example

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Homework
Review
Percentiles and Quartiles
Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

## Example (Quartiles)

- Find the median and quartiles of the following sample.

$$
5,8,10,15,17,19,20,24,25,30,32
$$



Median

## Example

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Homework
Review
Percentiles and Quartiles

Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

## Example (Quartiles)

- Find the median and quartiles of the following sample.

$$
5,8,10,15,17, \underset{\substack{19}}{(20,24,25,30,32}
$$

## Example

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles

Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

## Example (Quartiles)

- Find the median and quartiles of the following sample.

$$
5,8, \underset{\sim}{10}, 15,17, \underset{Q_{1}}{19}, 20,24, \underset{\text { Median }}{25}, 30,32
$$

## Definition (Five-number summary)

The five-number summary of a set of numbers consists of the five quantities

- Minimum
- $1^{\text {st }}$ quartile
- Median
- $3^{\text {rd }}$ quartile
- Maximum
- These five numbers divide the set of numbers into four groups of equal size, each containing one-fourth of the set.


## Example

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Homework
Review
Percentiles and Quartiles Example

## The

Five-Number Summary
Examples

## Example (Five-number summary)

- The five-number summary of the previous sample is
- $\mathrm{Min}=5$.
- $\mathrm{Q}_{1}=10$.
- $\mathrm{Med}=19$.
- $Q_{3}=25$.
- $\mathrm{Max}=32$.



## Practice

## Practice

- Find the five-number summary of the sample

$$
5,8,10,15,17,19,20,24,25,30,32,35 .
$$

## TI-83 Five-Number Summary

The Five-Number Summary

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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary Examples

## TI-83 Five-Number Summary

- Follow the same procedure that was used to find the mean.
- When the list of statistics appears, scroll down to the ones labeled

$$
\min X, ~ Q 1, ~ M e d, ~ Q 3, ~ m a x X . ~
$$

- They are the five-number summary.


## Tl-83 Five-Number Summary

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Homework Review

Percentiles and Quartiles Example

## The

Five-Number
Summary Examples

TI-83
Five-Number
Summary

## The

Interquartile Range

Percentiles in Excel

Assignment

## Tl-83 Five-number summary

- Use the TI-83 to find the five-number summary of the rainfall data

| 2.82 | 24.18 | 0.20 | 15.60 | 22.04 | 7.44 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| 5.16 | 9.14 | 37.36 | 10.19 | 2.16 | 17.50 |
| 28.12 | 11.23 | 8.66 | 7.24 | 6.50 | 4.88 |
| 13.08 | 4.01 | 11.28 | 1.96 | 12.09 | 2.92 |
| 7.67 | 4.39 | 6.60 | 6.50 | 25.43 | 0.74 |

## Five-Number Summaries and Distributions

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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

- If the distribution were uniform from 0 to 10 , what would be the five-number summary?



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The
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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

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## Five-Number Summaries and Distributions

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary
Examples
TI-83
Five-Number Summary

The
Interquartile
Range
Percentiles in Excel

Assignment

- If the distribution were uniform from 0 to 10 , what would be the five-number summary?



## Five-Number Summaries and Distributions

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

- Where would the median and quartiles be in this symmetric non-uniform distribution?



## Five-Number Summaries and Distributions

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

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- Where would the median and quartiles be in this symmetric non-uniform distribution?



## Five-Number Summaries and Distributions

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

- Where would the median and quartiles be in this symmetric non-uniform distribution?



## Five-Number Summaries and Distributions

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles
Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

- Describe the distribution.



## Five-Number Summaries and Distributions

The
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Homework
Review
Percentiles and Quartiles
Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

- Describe the distribution.



## Five-Number Summaries and Distributions

The
Five-Number Summary

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Homework
Review
Percentiles and Quartiles
Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

- Describe the distribution.



## Five-Number Summaries and Distributions

The
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Homework
Review
Percentiles and Quartiles
Example
The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

- Describe the distribution.



## The Interquartile Range

## Definition (Interquartile range)

The interquartile range, denoted IQR, is the difference between $\mathrm{Q}_{3}$ and $\mathrm{Q}_{1}$.

- The IQR is a commonly used measure of spread, or variability.
- Like the median, it is not affected by extreme outliers.


## The IQR

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Homework Review

Percentiles and Quartiles Example

## The

Five-Number
Summary
Examples
T1-83
Five-Number
Summary
The
Interquartile Range

Percentiles in Exce

Assignment

## Example (IQR)

- The IQR of

$$
5,8,10,15,17,19,20,24,25,30,32
$$

is

$$
\begin{aligned}
\text { IQR } & =Q_{3}-Q_{1} \\
& =25-10 \\
& =15
\end{aligned}
$$

## The IQR

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Homework Review

Percentiles and Quartiles Example

## The

Five-Number
Summary
Examples
TI-83
Five-Number Summary

The
Interquartile Range

Percentiles in Excel

Assignment

## Example (IQR)

- The IQR of

$$
5,8,10,15,17,19,20,24,25,30,32,35
$$

is

$$
\begin{aligned}
\text { IQR } & =Q_{3}-Q_{1} \\
& =27.5-12.5 \\
& =15
\end{aligned}
$$

## The IQR

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Homework
Review
Percentiles and Quartiles Example

The
Five-Number
Summary
Examples
TI-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

## Example (IQR)

- The IQR of the rainfall data is is

$$
\begin{aligned}
\text { IQR } & =Q_{3}-Q_{1} \\
& =13.08-4.39 \\
& =8.69 \mathrm{~cm}
\end{aligned}
$$

## The IQR

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Five-Number Summary

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Homework
Review
Percentiles and Quartiles

Example
The
Five-Number
Summary
Examples
T1-83
Five-Number
Summary
The
Interquartile
Range
Percentiles in Excel

Assignment

## Practice

- Find the five-number summary and the IQR of the sample

$$
5,20,30,45,60,80,100,140,175,200,240 .
$$

- Are the data skewed?


## Salaries of School Board Chairmen

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## Practice

- Find the five-number summary of the following salaries of school board chairmen.

| County/City | Salary | County/City | Salary |
| :--- | ---: | :--- | ---: |
| Henrico | 20,000 | Caroline | 5,000 |
| Chesterfield | 18,711 | Louisa | 4,921 |
| Richmond | 11,000 | Powhatan | 4,800 |
| Hanover | 11,000 | Hopewell | 4,500 |
| Petersburg | 8,500 | Charles City | 4,500 |
| Sussex | 7,000 | Prince George | 3,750 |
| New Kent | 6,500 | Cumberland | 3,600 |
| Goochland | 5,500 | King \& Queeen | 3,000 |
| Dinwiddie | 5,120 | King William | 2,400 |
| Colonial Hgts | 5,100 | West Point | 0 |

## Five-Number Summaries and Stem-and-Leaf Displays

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Homework Review

Percentiles and Quartiles Example

The
Five-Number
Summary

- It is possible to use a stem-and-leaf display to find a five-number summary, especially if the leaves are arranged in order.
- Find a five-number summary of the following January rainfall data.

| Stem | Leaf |
| ---: | :--- |
| 0 | 001222444 |
| 0 | 566677789 |
| 1 | 01123 |
| 1 | 57 |
| 2 | 24 |
| 2 | 58 |
| 3 |  |
| 3 | 7 |

- Note: 1|2 means 12.


## Excel's Definition of Percentile

- Microsoft's Excel uses a definition of the $p^{\text {th }}$ percentile that is based on the gaps between the numbers rather than on the numbers themselves.


## Definition (Excel's $p^{\text {th }}$ percentile)

Excel's $p^{\text {th }}$ percentile of a set of numbers is the number whose rank (position) is given by

$$
r=1+\left(\frac{p}{100}\right)(n-1)
$$

If $r$ is not a whole number, then interpolate between values.

## Assignment

## Homework

- Read Section 5.3.1-5.3.2, pages 312-315.
- Work Example 5.4, page 314, as an exercise.

