The Language of Statistical Decision Making

Lecture 2
Section 1.3

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Outline

1. Populations and Samples
2. Inferences
3. Hypotheses
4. Statistical Significance
5. Assignment
6. Answers to Even-Numbered Problems
**Definition (Population)**

The *population* is the set of all individuals *of interest* to the study.

**Definition (Sample)**

A *sample* is a set of individuals in the population from which data were obtained. Their characteristics were observed or measured.
Inferences

Definition (Statistical inference)

A statistical inference is a conclusion that is drawn about the population, based on what was observed in the sample.
Samples and Inferences

Population
Samples and Inferences

Population

Select a sample

Sample
Samples and Inferences

Population

Select a sample

Sample

Make the observations

Data
Samples and Inferences

Population

Select a sample

Sample

Make the observations

Data

Inference

Draw an inference
Samples and Inferences

Population

Sample

Data

Inference

Select a sample

Make the observations

Refers to the population

Draw an inference
Population of all possible coin tosses
Samples and Inferences

Population of all possible coin tosses

Select a sample

12 tosses
Samples and Inferences

Population of all possible coin tosses

Select a sample

12 tosses

Make the observations

8 heads
4 tails
Samples and Inferences

Population of all possible coin tosses

Select a sample

12 tosses

Make the observations

8 heads 4 tails

The coin is fair

Draw an inference

The coin is fair

12 tosses
Samples and Inferences

Population of all possible coin tosses

Select a sample

12 tosses

Make the observations

8 heads
4 tails

The coin is fair

Draw an inference

Refers to the population

The coin is fair

Refers to the population

Make the observations

12 tosses

Select a sample

Population of all possible coin tosses
Example (Samples and Inferences)

- Is the earth getting warmer?
- How can we tell?
All points on the earth's surface (Population)
Samples and Inferences

Example (Samples and Inferences)

- All points on the earth's surface (Population)
- Select a sample
- A few points (Sample)
Samples and Inferences

Example (Samples and Inferences)

All points on the earth's surface (Population)

Select a sample

A few points (Sample)

Measure the temperature

Temperature readings (Data)
Samples and Inferences

Example (Samples and Inferences)

All points on the earth's surface (Population)

Select a sample

A few points (Sample)

Measure the temperature

Temperature readings (Data)

The earth is getting warmer (Inference)

Draw an inference

A few points (Sample)

Select a sample

Temperature readings (Data)

The earth is getting warmer (Inference)

Draw an inference
Samples and Inferences

Example (Samples and Inferences)

- All points on the earth's surface (Population)
  - Refers to the entire earth
  - Select a sample
    - A few points (Sample)
    - Measure the temperature
    - Temperature readings (Data)
  - Draw an inference
    - The earth is getting warmer (Inference)

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Hypotheses

Definition (Hypothesis)

A hypothesis is a statement that is proposed to explain an observed phenomenon.

- In statistics the “phenomenon” is what was observed in the sample.
- The hypothesis is a statement about the population that explains the phenomenon.
- Typically, the hypothesis says that we observed what we did in the sample because it is true for the entire population.
Example (The Coin-Toss Experiment)

- When we tossed the dollar coin, it landed heads 8 out of 12 times.
- That was not 50% of the tosses. How do we explain it?
- One hypothesis could be that the coin is not fair, but it is biased towards heads. Perhaps the probability of heads is $\frac{2}{3}$.
- What is another hypothesis?
Hypotheses

Example (Global Warming)

- Suppose our temperature measurements of the earth are warmer on the average (than in the past).
- One hypothesis is that we observed warmer temperatures at the few places we observed because the earth is getting warmer.
- What is another hypothesis?
Example (Global Warming)

- In statistics we do not entertain the hypotheses that the earth is getting warmer because of increased CO$_2$ or because of human activity.
- Why not?
Hypotheses

Example (Global Warming)

Why are our sample points getting warmer?

Because the earth is getting warmer

Statistical hypothesis

Why is the earth getting warmer?

Because there is more CO$_2$ in the atmosphere

Non-statistical hypothesis
Null Hypothesis

Definition (Null Hypothesis)

The null hypothesis $H_0$ is the conventional belief about the population, or the status quo, or the neutral position.

- It requires no special justification.
- That is, it receives the benefit of the doubt.
Definition (Alternative (or Research) Hypothesis)

The alternative hypothesis $H_1$ is an alternative to the null hypothesis.

- It requires evidence for its justification.
- That is, it bears the burden of proof.
Population → Sample → Data

- Select a sample
- Make the observations

Null hypothesis
Alternative hypothesis
Population

Select a sample

Sample

Data

Null hypothesis

Alternative hypothesis

Make the observations

Which hypothesis is supported?
Population

Sample

Select a sample

Data

Null hypothesis

Alternative hypothesis

It may support the null hypothesis

Make the observations
Population

Sample

Select a sample

Make the observations

Data

Alternative hypothesis

Null hypothesis

It may support the alternative Null hypothesis
Null and Alternative Hypotheses

Example (Null and Alternative Hypotheses)

- In the case of global warming, what are the null and alternative hypotheses?
- What evidence would support the alternative hypothesis?
- What evidence would be *consistent with* the null hypothesis?
Null and Alternative Hypotheses

Example (Null and Alternative Hypotheses)

- All points on the surface of the earth
- A few points
- Select a sample
- Make the observations
- Higher temperatures
- Earth is warming
- Earth is not warming

Which hypothesis is supported?
Null and Alternative Hypotheses

Example (Null and Alternative Hypotheses)

- All points on the surface of the earth
- Select a sample
- A few points
- Make the observations
- Slightly higher temperatures
- Earth is warming
- Null hypothesis is supported
- Earth is not warming
Null and Alternative Hypotheses

Example (Null and Alternative Hypotheses)

- All points on the surface of the earth
- A few points
  - Select a sample
  - Make the observations
- Much higher temperatures
  - Alternative hypothesis is supported

Earth is warming

Earth is not warming
The data are called \textbf{statistically significant} if their deviation from what would be expected under the null hypothesis is too great to be attributed to sampling error.
In the global warming example, an increase in the sample of 0.1°F may not be statistically significant if it could be attributed to sampling error.

An increase of 5°F most likely would be statistically significant as it could not be attributed to chance (sampling error).
Example (Statistical Significance)

- If the earth is not getting warmer, then it may be likely that we would see an increase in the sample of 0.1°F (due to randomness).

- If the earth is not getting warmer, then it is very unlikely that we would see an increase of 5°F.
Assignment

Homework

- Read Section 1.3 - 1.3.2, pages 4 - 11.
- Let's Do It! 1.1, 1.2, 1.3, 1.4.
- Page 67, exercises 1 - 3.
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Page 67, Exercise 1.2

1.2 (a) true.
(b) true.
(c) false.
(d) false.