Hypothesis Testing for Means
Lecture 33
Sections 10.1-10.2

Robb T. Koether
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Outline

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3. Hypothesis Testing for the Mean
   - The Hypotheses
   - The Level of Significance
   - The Test Statistic
   - The Value of the Test Statistic
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   - The Conclusion
4. Another Example
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6. Assignment
A new method for detecting a type of cancer has been developed. Among 80 adults who have this type of cancer, this method failed to detect the cancer in five of the adults. Provide a 92% confidence interval estimate for the failure rate for this method.
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- For a 92% C.I., \( \alpha = 0.08 \), so \( \alpha/2 = 0.04 \).
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- For a 92% C.I., $\alpha = 0.08$, so $\alpha/2 = 0.04$.
- The coefficient is $z_{0.04} = \text{invNorm}(0.04) = -1.751$. 

The confidence interval is $\hat{p} \pm z_{0.04} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.0625 \pm (1.751)\left(\frac{0.0625 \cdot 0.9375}{80}\right) = 0.0625 \pm 0.0474$. 

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- For a 92% C.I., \( \alpha = 0.08 \), so \( \alpha/2 = 0.04 \).
- The coefficient is \( z_{0.04} = \text{invNorm}(0.04) = -1.751 \).
- So, the confidence interval is

\[
\hat{p} \pm z_{0.04} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.0625 \pm 1.751 \sqrt{\frac{(0.0625)(0.9375)}{80}} \\
= 0.0625 \pm (1.751)(0.0271) \\
= 0.0625 \pm 0.0474.
\]
Solution

- Or you could use the TI-83 function $1 -$ PropZInt.
- Enter
  - $x = 5$.
  - $n = 80$.
  - $C$-Level $= 0.92$.
- The calculator reports the interval $(0.01512, 0.10988)$. 
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6. Assignment
We now ask the two basic questions, this time about the mean.
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In most ways, Chapter 10 will be like Chapter 9.
Introduction

- We now ask the two basic questions, this time about the mean.
  - Is a given hypothesis concerning $\mu$ true?
  - What is the value of $\mu$?
- In most ways, Chapter 10 will be like Chapter 9.
- In one way, it will be very different.
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6. Assignment
The Seven Steps of Hypothesis Testing

1. State the null and alternative hypotheses.
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1. State the null and alternative hypotheses.
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7. State the conclusion.
An Example

- In an attempt to determine whether the price for a gallon of regular gasoline is less than $2.70, a reporter samples 36 service stations.
- He finds an average price of $2.63.
- The population standard deviation $\sigma$ is (somehow) known to be $0.12$.
- At the 5% level of significance, test the hypothesis that the average price of a gallon of gasoline is less than $2.70$. 
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6. Assignment
The Hypotheses

- Tell what $\mu$ represents.
- The null hypothesis gives a hypothetical value $\mu_0$ for the population mean.
  \[ H_0 : \mu = \mu_0. \]
- The alternative hypothesis contradicts $H_0$ in one of three ways.
  - $H_1 : \mu < \mu_0$.
  - $H_1 : \mu > \mu_0$.
  - $H_1 : \mu \neq \mu_0$. 

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The Hypotheses

Example (Step 1)

(1) Let $\mu$ represent the average price of a gallon of regular gas.

$H_0 : \mu = 2.70$

$H_1 : \mu < 2.70$
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The Level of Significance

- State the level of significance (the value of $\alpha$).
Example (Step 2)

(2) Let $\alpha = 0.05$. 
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6. Assignment
The Test Statistic

- The choice of test statistic is where things get a little complicated.
- The choice will depend on the sample size and what is known about the population. (Details to follow.)
- If we assume that $\sigma$ is known and that either
  - The sample size $n$ is at least 30, or
  - The population is normal,
then the Central Limit Theorem for Means will apply, telling us that $\bar{x}$ is normal with mean $\mu_0$ (if $H_0$ is true) and standard deviation $\frac{\sigma}{\sqrt{n}}$. 
The Sampling Distribution of $\bar{x}$

Therefore, the test statistic, for now, is

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$
The Decision Tree

Is \( \sigma \) known?

- yes
- no
The Decision Tree

Is $\sigma$ known?

- yes
- no

Is the population normal?

- yes
- no

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The Decision Tree

Is $\sigma$ known?

- yes
  - Is the population normal?
    - yes
      - $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$
    - no

- no
  - Is the population normal?
    - yes
    - no
The Decision Tree

Is σ known?

- yes
  - Is the population normal?
    - yes
      - \( Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \)
    - no
  - no

- no
  - Is \( n \geq 30? \)
    - yes
    - no
The Decision Tree

Is $\sigma$ known?

Is the population normal?

Is $n \geq 30$?

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$Z \approx \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$
The Decision Tree

Is $\sigma$ known?

Is the population normal?

yes

no

Is $n \geq 30$?

yes

no

Give up

$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

$Z \approx \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$
The Decision Tree

Is $\sigma$ known?

yes

Is the population normal?

yes

$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

no

Come back later

no

Is $n \geq 30$?

yes

$Z \approx \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

no

Give up
Example (Step 3)

(3) Let the test statistic be

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$
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6. Assignment
Substitute the values of $\bar{x}$, $\mu_0$, $\sigma$, and $n$ into the formula to get the value of $z$. 
Example (Step 4)

(4)  

\[ \mu_0 = 2.70. \]
\[ \bar{x} = 2.63. \]
\[ \sigma = 0.12. \]
\[ n = 36. \]

Therefore, 

\[ z = \frac{2.63 - 2.70}{0.12/\sqrt{36}} = \frac{-0.07}{0.02} = -3.500. \]
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The $p$-Value

- To find the $p$-value, use `normalcdf` on the TI-83.
- For one-tailed tests, compute either
  - $p$-value = `normalcdf(-E99, z)`, or
  - $p$-value = `normalcdf(z, E99)`.
- For a two-tailed test, compute the area of the appropriate tail, and then double it.
Example (Step 5)

(5) \( p\)-value = \text{normalcdf}(-E99, -3.5) = 2.328 \times 10^{-4}.\)
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The Decision

- Reject the null hypothesis if the $p$-value is less than $\alpha$.
- Otherwise, accept it.
- Write either “Reject $H_0$” or “Accept $H_0$.”
Example (Step 6)

(6) Reject $H_0$. 

The Decision
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6. Assignment
State the conclusion in plain English, using the terminology of the original problem, not the statistical jargon.
Example (Step 7)

(7) The average price of a gallon of regular gasoline is less than $2.70.
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Exercise

Let’s Do It! 10.2, page 620.

- Under normal circumstances, mice complete a maze in an average time of 18 seconds, with a standard deviation of 2 seconds.
- A researcher introduces loud noises. Will this cause the mice to run the maze faster?
- A sample of 10 mice has an average time of 17 seconds.
- Assume that their run times are normally distributed.
- The population standard deviation is known to be 2 seconds.
- Test the hypothesis at the 10% level that the average run time is less when there is loud noise.
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**TI-83 Hypothesis Testing for the Mean**

- **Press** `STAT`.
- **Select** `TESTS`.
- **Select** `Z-Test`.
- **Press** `ENTER`. A window appears requesting information.
- **Select** `Data` if you have the sample data entered into a list.
- **Otherwise, select** `Stats`. 
Hypothesis Testing on the TI-83

The Stats Option

- Enter $\mu_0$, the hypothetical mean.
- Enter $\sigma$. (Remember, $\sigma$ is known.)
- Enter $\bar{x}$.
- Enter $n$, the sample size.
- Select the type of alternative hypothesis.
- Select Calculate and press ENTER.
Hypothesis Testing on the TI-83
The Stats Option

TI-83 Hypothesis Testing for the Mean

- A window appears with the following information.
  - The title Z-Test.
  - The alternative hypothesis.
  - The value of the test statistic $Z$.
  - The $p$-value of the test.
  - The sample mean.
  - The sample size.
Assignment

Homework

- Read Sections 10.1 - 10.2, pages 613 - 620.
- Let’s Do It! 10.1, 10.2.
- Exercises 1 - 6, page 633.
2. $H_0 : \mu = 7$ vs. $H_1 : \mu < 7$.
$\mu$ represents the population mean time to complete the maze.

4. $z = 2.8$ and $p$-value $= 0.002555$. Reject $H_0$. The average IQ of girls at the alternative school is greater than 100.
6. (a) (Step 1) $H_0 : \mu = 32$ vs. $H_1 : \mu > 32$.
(b) (Steps 2 - 6.)

(2) $\alpha = 0.025$.
(3) $Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$.
(4) $z = 8.275$.
(5) $p$-value $= 6.449 \times 10^{-17}$.
(6) Reject $H_0$.

(c) (Step 7) If the calories in their diet are replaced by vitamins and protein, then on the average mice will live longer than 32 months.