# Logarithmic Scales   Math 121 - Workshop

Seeing there is nothing that is so troublesome to mathematical practice.... than the multiplications, divisions, square and cubical extractions of great numbers, which besides the tedious expense of time are... subject to many slippery errors, I began therefore to consider [how] I might remove those hindrances.

                                                            - John Napier, 1614

The line below is called a *logarithmic scale.* Unlike a normal ruler, the numbers are spaced according to multiplication & division instead of addition & subtraction.

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1        2       3     4    5   6  7  8  9 10                              100
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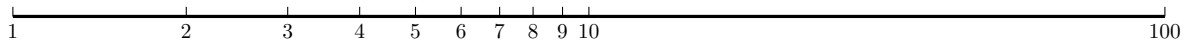
1. Use another piece of paper to measure and mark the place where 20 should be on the log-scale.

2. Mark the positions of 36, 54, and 81 on the logarithmic scale.

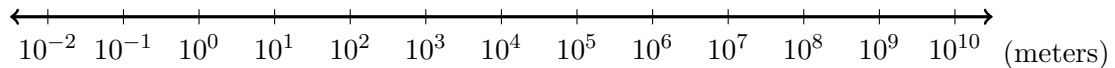3. Where does the fraction $\frac{5}{4}$ go on the logarithmic scale?

The following chart from the Economist magazine shows the correlation between urbanization and per capita income.
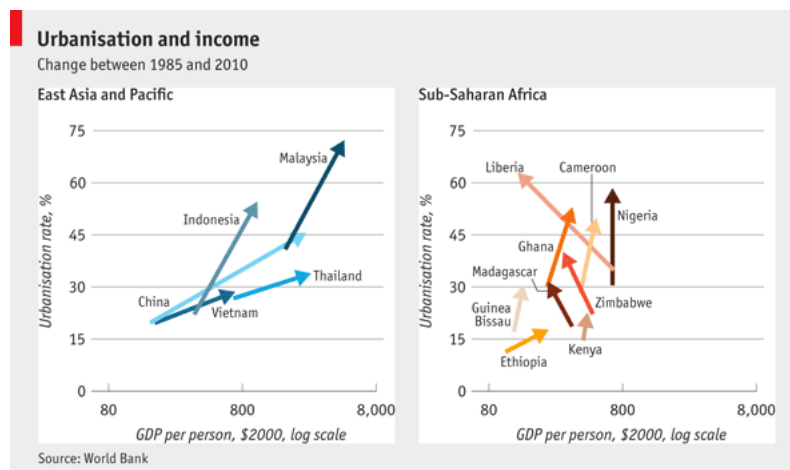
4. Which axis is log-scale? How can you tell?

5. Which country has seen its GDP per person grow by more than an order of magnitude?

6. Place the following values on the logarithmic scale below. Try to find the correct position between the orders of magnitude shown.

   (a) Height of an average person

   (b) Radius of Earth - 6,378 kilometers (be sure to convert to meters first!)

   (c) Radius of the Sun - about 100 times larger than the Earth

   (d) Length of a mouse - 8 centimeters

$$10^{-2} \quad 10^{-1} \quad 10^{0} \quad 10^{1} \quad 10^{2} \quad 10^{3} \quad 10^{4} \quad 10^{5} \quad 10^{6} \quad 10^{7} \quad 10^{8} \quad 10^{9} \quad 10^{10} \quad \text{(meters)}$$

7. Log-scales are the best choice for representing data spread over several different orders of magnitude. Which of the following data is likely to be spread over multiple orders of magnitude?

   (a) Heights of adult men in the United States.

   (b) Populations of all the counties in the U.S.

   (c) Incomes of American citizens.

   (d) Ages of American citizens.