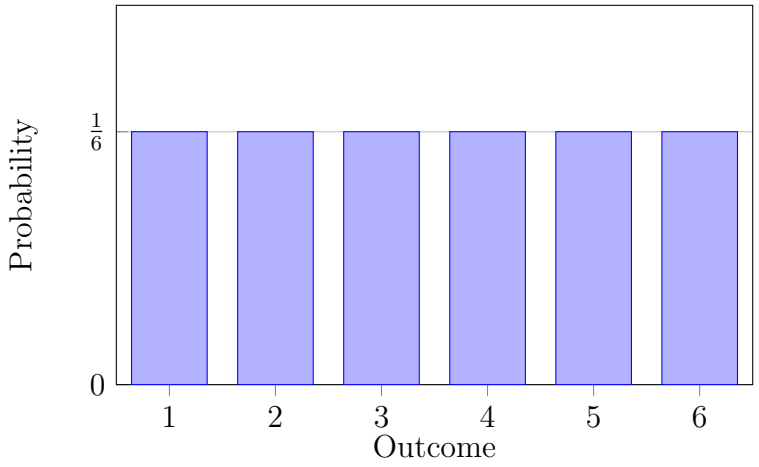
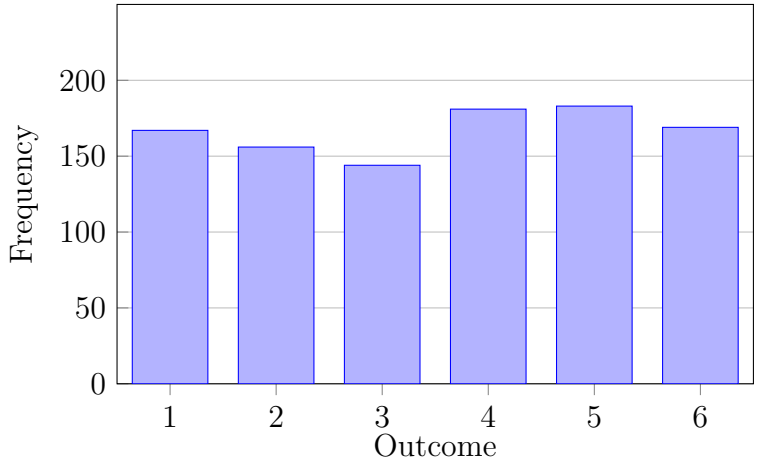
Example 1: 6-Sided Die

A comparison of the theoretical probability model of a random variable versus a simulation.

Probability Model



Actual Distribution of a Sample ($N = 1000$)



Theoretical Parameters

- Theoretical Average

$$E(X) = 3.5$$

- Theoretical Variance

$$\text{Var}(X) = 2.917$$

The theoretical standard deviation is $\sigma = \sqrt{2.917} = 1.708$.

Sample Statistics

- Sample Mean

$$\bar{x} = 3.564$$

- Sample Variance

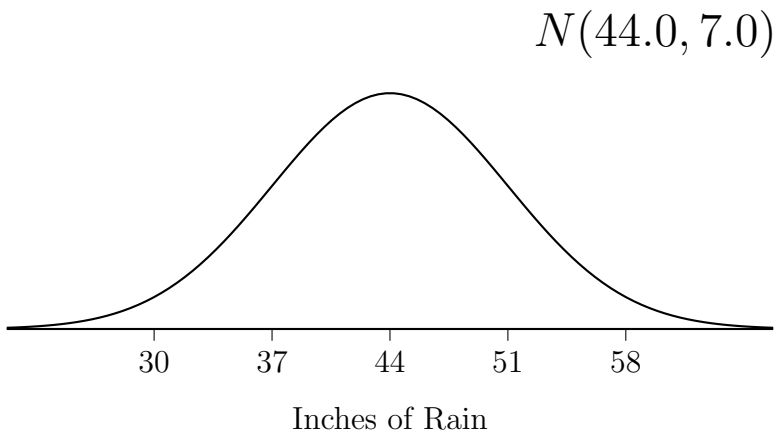
$$s^2 = 2.943$$

The sample standard deviation is $s = \sqrt{2.943} = 1.7155$.

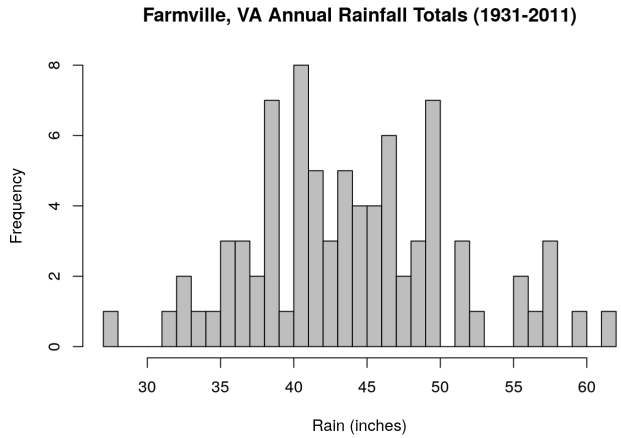
Example 2: Annual Rainfall in Farmville

A comparison of the theoretical probability model of a random variable versus a simulation.

Probability Model



Actual Distribution of a Sample



Theoretical Parameters

- Theoretical Average

$$E(X) = 44.0 \text{ inches}$$

- Theoretical Variance

$$\text{Var}(X) = 49.0$$

The theoretical standard deviation is $\sigma = \sqrt{49.0} = 7.0$ inches.

Sample Statistics

- Sample Mean

$$\bar{x} = 44.02 \text{ inches}$$

- Sample Variance

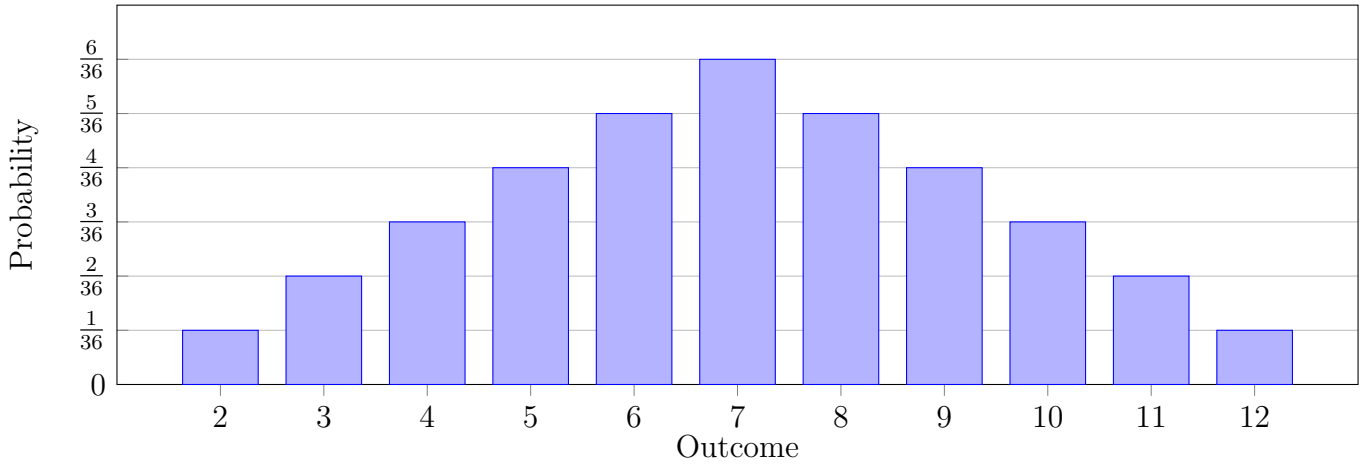
$$s^2 = 48.72$$

So the sample standard deviation is $s = \sqrt{48.72} = 6.98$ inches.

Adding Random Variables

If you add two random variables together, you get a new random variable. For example, if X and Y both represent the results of different six-sided dice, then $X + Y$ is their combined total.

Probability Distribution for $X + Y$



Rules for Adding Random Variables

- $\mu_{X+Y} = \mu_X + \mu_Y$ (always)
- $\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$ (if X and Y are independent)