# **Common Inference Formulas in Statistics**

## Inference about Means

### **One Sample Hypothesis Test**

$$\begin{array}{l} H_0: \mu = \mu_0 \\ H_A: \mu \neq \mu_0 \end{array} \qquad t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \end{array}$$

#### Two Sample Hypothesis Test

$$\begin{array}{l} H_0: \mu_A = \mu_B \\ H_A: \mu_A \neq \mu_B \end{array} \qquad t = \frac{\bar{x}_A - \bar{x}_B}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} \end{array}$$

## **One Sample Confidence Interval**

 $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ 

## **Two Sample Confidence Interval**

$$\bar{x}_A - \bar{x}_B \pm t^* \sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}$$

These formulas are robust even if the population is not normal, as long as the sample is "nice" ( $n \ge 40$  usually safe;  $n \ge 15$  okay for samples w/ little skew & no outliers). Two sample *t*-distribution methods are usually more robust than one sample methods. If the two samples are similar sizes, then you can add the sizes to evaluate robustness ( $n_A + n_B \ge 15$  might be okay,  $n_A + n_B \ge 40$  is better).

## **Inference about Proportions**

## **One Sample Hypothesis Test**

$$\begin{array}{l} H_0: p = p_0 \\ H_A: p \neq p_0 \end{array} \qquad z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}} \end{array}$$

This is robust, as long as there are at least 10 expected successes and 10 expected failures in the sample. Otherwise, you might use a binomial model.

#### Two Sample Hypothesis Test

$$\begin{array}{l} H_0: p_A = p_B \\ H_A: p_A \neq p_B \end{array} \qquad z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} \end{array}$$

where  $\hat{p}$  is the pooled proportion. This is fairly robust, as long as their are at least 5 successes and 5 failures in each sample. Otherwise, you can use a permutation test instead.

## **One Sample Confidence Interval**

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Confidence intervals for proportions are less robust than hypothesis tests (because you are using  $\hat{p}$  instead of  $p_0$  to estimate the standard error). On the other hand, plus-4 confidence intervals are actually more robust than hypothesis tests. To make a plus-4 confidence interval, add 2 fake successes and 2 fake failures to the sample.

### **Two Sample Confidence Interval**

$$\hat{p}_A - \hat{p}_B \pm z^* \sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n_A} + \frac{\hat{p}_B(1-\hat{p}_B)}{n_B}}$$

To make a plus-4 confidence interval for two sample proportions, add 1 success and 1 failure to each sample.

## Inference about Other Statistics

Other common inference techniques include the  $\chi^2$  tests for association and goodness of fit, ANOVA F-tests, and various methods related to linear regression. Another tool is bootstrapping, which can be applied to any statistic.