4.2 EXERCISES

46. What is the linear approximation for any generic linear function $y = mx + b$?

47. Determine the necessary conditions such that the linear approximation function is constant. Use a graph to prove your result.

48. Explain why the linear approximation becomes less accurate as you increase the distance between x and a. Use a graph to prove your argument.

49. When is the linear approximation exact?

For the following exercises, find the linear approximation $L(x)$ to $y = f(x)$ near $x = a$ for the function.

50. **[T]** $f(x) = x + x^4,\ a = 0$

51. **[T]** $f(x) = \frac{1}{x},\ a = 2$

52. **[T]** $f(x) = \tan x,\ a = \frac{\pi}{4}$

53. **[T]** $f(x) = \sin x,\ a = \frac{\pi}{2}$

54. **[T]** $f(x) = x \sin x,\ a = 2\pi$

55. **[T]** $f(x) = \sin^2 x,\ a = 0$

For the following exercises, compute the values given within 0.01 by deciding on the appropriate $f(x)$ and a, and evaluating $L(x) = f(a) + f'(a)(x - a)$. Check your answer using a calculator.

56. **[T]** $(2.001)^6$

57. **[T]** $\sin(0.02)$

58. **[T]** $\cos(0.03)$

59. **[T]** $(15.99)^{1/4}$

60. **[T]** $\frac{1}{0.98}$

61. **[T]** $\sin(3.14)$

For the following exercises, determine the appropriate $f(x)$ and a, and evaluate $L(x) = f(a) + f'(a)(x - a)$. Calculate the numerical error in the linear approximations that follow.

62. $(1.01)^3$

63. $\cos(0.01)$

64. $(\sin(0.01))^2$

65. $(1.01)^{-3}$

66. $\left(1 + \frac{1}{10}\right)^{10}$

67. $\sqrt{8.99}$

For the following exercises, find the differential of the function.

68. $y = 3x^4 + x^2 - 2x + 1$

69. $y = x \cos x$

70. $y = \sqrt{1 + x}$

71. $y = \frac{x^2 + 2}{x - 1}$

For the following exercises, find the differential and evaluate for the given x and dx.

72. $y = 3x^2 - x + 6,\quad x = 2,\quad dx = 0.1$

73. $y = \frac{1}{x + 1},\quad x = 1,\quad dx = 0.25$

74. $y = \tan x,\quad x = 0,\quad dx = \frac{\pi}{10}$

75. $y = \frac{3x^2 + 2}{\sqrt{x + 1}},\quad x = 0,\quad dx = 0.1$

76. $y = \frac{\sin(2x)}{x},\quad x = \pi,\quad dx = 0.25$

77. $y = x^3 + 2x + \frac{1}{x},\quad x = 1,\quad dx = 0.05$

For the following exercises, find the change in volume dV or in surface area dA.

78. dV if the sides of a cube change from 10 to 10.1.

79. dA if the sides of a cube change from x to $x + dx$.

80. dA if the radius of a sphere changes from r by dr.