

4.7 EXERCISES

For the following exercises, answer by proof, counterexample, or explanation.

311. When you find the maximum for an optimization problem, why do you need to check the sign of the derivative around the critical points?

312. Why do you need to check the endpoints for optimization problems?

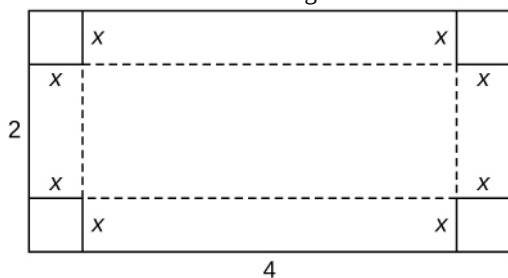
313. *True or False.* For every continuous nonlinear function, you can find the value x that maximizes the function.

314. *True or False.* For every continuous nonconstant function on a closed, finite domain, there exists at least one x that minimizes or maximizes the function.

For the following exercises, set up and evaluate each optimization problem.

315. To carry a suitcase on an airplane, the length + width + height of the box must be less than or equal to 62 in. Assuming the height is fixed, show that the maximum volume is $V = h\left(31 - \left(\frac{1}{2}\right)h\right)^2$. What height allows you to have the largest volume?

316. You are constructing a cardboard box with the dimensions 2 m by 4 m. You then cut equal-size squares from each corner so you may fold the edges. What are the dimensions of the box with the largest volume?



317. Find the positive integer that minimizes the sum of the number and its reciprocal.

318. Find two positive integers such that their sum is 10, and minimize and maximize the sum of their squares.

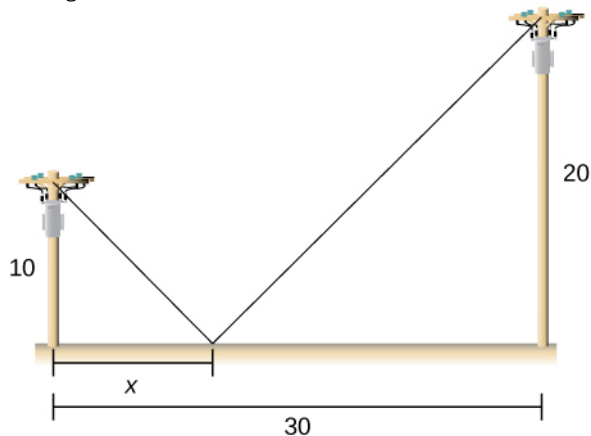
For the following exercises, consider the construction of a pen to enclose an area.

319. You have 400 ft of fencing to construct a rectangular pen for cattle. What are the dimensions of the pen that maximize the area?

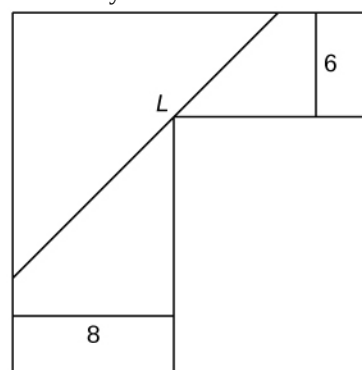
320. You have 800 ft of fencing to make a pen for hogs. If you have a river on one side of your property, what is the dimension of the rectangular pen that maximizes the area?

321. You need to construct a fence around an area of 1600 ft. What are the dimensions of the rectangular pen to minimize the amount of material needed?

322. Two poles are connected by a wire that is also connected to the ground. The first pole is 20 ft tall and the second pole is 10 ft tall. There is a distance of 30 ft between the two poles. Where should the wire be anchored to the ground to minimize the amount of wire needed?



323. **[T]** You are moving into a new apartment and notice there is a corner where the hallway narrows from 8 ft to 6 ft. What is the length of the longest item that can be carried horizontally around the corner?



324. A patient's pulse measures 70 bpm, 80 bpm, then 120 bpm. To determine an accurate measurement of pulse, the doctor wants to know what value minimizes the expression $(x - 70)^2 + (x - 80)^2 + (x - 120)^2$? What value minimizes it?