4.8 EXERCISES

For the following exercises, evaluate the limit.

356. Evaluate the limit $\lim_{x \to \infty} \frac{e^x}{x}$.

357. Evaluate the limit $\lim_{x \to \infty} \frac{e^x}{x^k}$.

358. Evaluate the limit $\lim_{x \to \infty} \frac{\ln x}{x^k}$.

359. Evaluate the limit $\lim_{x \to a} \frac{x-a}{x^2-a^2}$, $\quad a \neq 0$.

360. Evaluate the limit $\lim_{x \to a} \frac{x-a}{x^3-a^3}$, $\quad a \neq 0$.

361. Evaluate the limit $\lim_{x \to a} \frac{x-a}{x^n-a^n}$, $\quad a \neq 0$.

For the following exercises, determine whether you can apply L'Hôpital's rule directly. Explain why or why not. Then, indicate if there is some way you can alter the limit so you can apply L'Hôpital's rule.

362. $\lim_{x \to 0^+} x^2 \ln x$

363. $\lim_{x \to \infty} x^{1/x}$

364. $\lim_{x \to 0} x^{2/x}$

365. $\lim_{x \to 0} \frac{x^2}{1/x}$

366. $\lim_{x \to \infty} \frac{e^x}{x}$

For the following exercises, evaluate the limits with either L'Hôpital's rule or previously learned methods.

367. $\lim_{x \to 3} \frac{x^2-9}{x-3}$

368. $\lim_{x \to 3} \frac{x^2-9}{x+3}$

369. $\lim_{x \to 0} \frac{(1+x)^{-2}-1}{x}$

370. $\lim_{x \to \pi/2} \frac{\cos x}{\frac{\pi}{2}-x}$

371. $\lim_{x \to \pi} \frac{x-\pi}{\sin x}$

372. $\lim_{x \to 1} \frac{x-1}{\sin x}$

373. $\lim_{x \to 0} \frac{(1+x)^n-1}{x}$

374. $\lim_{x \to 0} \frac{(1+x)^n-1-nx}{x^2}$

375. $\lim_{x \to 0} \frac{\sin x - \tan x}{x^3}$

376. $\lim_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

377. $\lim_{x \to 0} \frac{e^x-x-1}{x^2}$

378. $\lim_{x \to 0} \frac{\tan x}{\sqrt{x}}$

379. $\lim_{x \to 1} \frac{x-1}{\ln x}$

380. $\lim_{x \to 0} (x+1)^{1/x}$

381. $\lim_{x \to 1} \frac{\sqrt{x}-\sqrt[3]{x}}{x-1}$

382. $\lim_{x \to 0^+} x^{2x}$

383. $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

384. $\lim_{x \to 0} \frac{\sin x - x}{x^2}$

385. $\lim_{x \to 0^+} x \ln\left(x^4\right)$

386. $\lim_{x \to \infty} (x - e^x)$

387. $\lim_{x \to \infty} x^2 e^{-x}$

388. $\lim_{x \to 0} \frac{3^x-2^x}{x}$

389. $\lim_{x \to 0} \frac{1+1/x}{1-1/x}$

390. $\lim_{x \to \pi/4} (1 - \tan x) \cot x$