4.9 EXERCISES

For the following exercises, write Newton's formula as $x_{n+1} = F(x_n)$ for solving $f(x) = 0$.

406. $f(x) = x^2 + 1$

407. $f(x) = x^3 + 2x + 1$

408. $f(x) = \sin x$

409. $f(x) = e^x$

410. $f(x) = x^3 + 3xe^x$

For the following exercises, solve $f(x) = 0$ using the iteration $x_{n+1} = x_n - cf(x_n)$, which differs slightly from Newton's method. Find a c that works and a c that fails to converge, with the exception of $c = 0$.

411. $f(x) = x^2 - 4$, with $x_0 = 0$

412. $f(x) = x^2 - 4x + 3$, with $x_0 = 2$

413. What is the value of "c" for Newton's method?

For the following exercises, start at

a. $x_0 = 0.6$ and

b. $x_0 = 2$.

Compute x_1 and x_2 using the specified iterative method.

414. $x_{n+1} = {x_n}^2 - \frac{1}{2}$

415. $x_{n+1} = 2x_n(1 - x_n)$

416. $x_{n+1} = \sqrt{x_n}$

417. $x_{n+1} = \frac{1}{\sqrt{x_n}}$

418. $x_{n+1} = 3x_n(1 - x_n)$

419. $x_{n+1} = {x_n}^2 + x_n - 2$

420. $x_{n+1} = \frac{1}{2}x_n - 1$

421. $x_{n+1} = |x_n|$

For the following exercises, solve to four decimal places using Newton's method and a computer or calculator. Choose any initial guess x_0 that is not the exact root.

422. $x^2 - 10 = 0$

423. $x^4 - 100 = 0$

424. $x^2 - x = 0$

425. $x^3 - x = 0$

426. $x + 5\cos(x) = 0$

427. $x + \tan(x) = 0$, choose $x_0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

428. $\frac{1}{1 - x} = 2$

429. $1 + x + x^2 + x^3 + x^4 = 2$

430. $x^3 + (x + 1)^3 = 10^3$

431. $x = \sin^2(x)$

For the following exercises, use Newton's method to find the fixed points of the function where $f(x) = x$; round to three decimals.

432. $\sin x$

433. $\tan(x)$ on $x = \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

434. $e^x - 2$

435. $\ln(x) + 2$

Newton's method can be used to find maxima and minima of functions in addition to the roots. In this case apply Newton's method to the derivative function $f'(x)$ to find its roots, instead of the original function. For the following exercises, consider the formulation of the method.

436. To find candidates for maxima and minima, we need to find the critical points $f'(x) = 0$. Show that to solve for the critical points of a function $f(x)$, Newton's method is given by $x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$.

437. What additional restrictions are necessary on the function f?