5.5 EXERCISES

254. Why is u-substitution referred to as *change of variable*?

255. 2. If $f = g \circ h$, when reversing the chain rule, $\frac{d}{dx}(g \circ h)(x) = g'(h(x))h'(x)$, should you take $u = g(x)$ or $u = h(x)$?

In the following exercises, verify each identity using differentiation. Then, using the indicated u-substitution, identify f such that the integral takes the form $\int f(u)du$.

256. $\int x\sqrt{x+1}dx = \frac{2}{15}(x+1)^{3/2}(3x-2) + C;\ u = x+1$

257. $\int \frac{x^2}{\sqrt{x-1}}dx\,(x > 1) = \frac{2}{15}\sqrt{x-1}\left(3x^2 + 4x + 8\right) + C;\ u = x-1$

258. $\int x\sqrt{4x^2+9}dx = \frac{1}{12}\left(4x^2+9\right)^{3/2} + C;\ u = 4x^2+9$

259. $\int \frac{x}{\sqrt{4x^2+9}}dx = \frac{1}{4}\sqrt{4x^2+9} + C;\ u = 4x^2+9$

260. $\int \frac{x}{(4x^2+9)^2}dx = -\frac{1}{8(4x^2+9)};\ u = 4x^2+9$

In the following exercises, find the antiderivative using the indicated substitution.

261. $\int (x+1)^4 dx;\ u = x+1$

262. $\int (x-1)^5 dx;\ u = x-1$

263. $\int (2x-3)^{-7} dx;\ u = 2x-3$

264. $\int (3x-2)^{-11} dx;\ u = 3x-2$

265. $\int \frac{x}{\sqrt{x^2+1}}dx;\ u = x^2+1$

266. $\int \frac{x}{\sqrt{1-x^2}}dx;\ u = 1-x^2$

267. $\int (x-1)\left(x^2-2x\right)^3 dx;\ u = x^2-2x$

268. $\int \left(x^2-2x\right)\left(x^3-3x^2\right)^2 dx;\ u = x^3 = 3x^2$

269. $\int \cos^3\theta d\theta;\ u = \sin\theta\ \ (Hint: \cos^2\theta = 1 - \sin^2\theta)$

270. $\int \sin^3\theta d\theta;\ u = \cos\theta\ \ (Hint: \sin^2\theta = 1 - \cos^2\theta)$

In the following exercises, use a suitable change of variables to determine the indefinite integral.

271. $\int x(1-x)^{99} dx$

272. $\int t\left(1-t^2\right)^{10} dt$

273. $\int (11x-7)^{-3} dx$

274. $\int (7x-11)^4 dx$

275. $\int \cos^3\theta\sin\theta d\theta$

276. $\int \sin^7\theta\cos\theta d\theta$

277. $\int \cos^2(\pi t)\sin(\pi t)dt$

278. $\int \sin^2 x\cos^3 x dx\ \ (Hint: \sin^2 x + \cos^2 x = 1)$

279. $\int t\sin\left(t^2\right)\cos\left(t^2\right)dt$

280. $\int t^2\cos^2\left(t^3\right)\sin\left(t^3\right)dt$

281. $\int \frac{x^2}{\left(x^3-3\right)^2}dx$

282. $\int \frac{x^3}{\sqrt{1-x^2}}dx$