Math 142 - Midterm 1 Recap Problems **Name:** ______________________

Each of these problems is worth 1% of the points you lost on Midterm 1. Take your time and do as many as you can. You may use a computer and/or ask me for help. There is no partial credit, so check your answers carefully. Due: Mon, Oct 8.

1. Find the area between the curves $y = x^2$ and $y = -x^2 + 18x$.

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2. Find the area of the region between $y = e^x$, $y = e^{2x-1}$, and $x = 0$.

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3. Solve $\ln x - \ln(x-2) = \ln 3$.

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4. Find the inverse of the function $f(x) = 1 - 2^{-x}$.

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5. Simplify $\log_2 400 - \log_2 25$.

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6. $\dfrac{d}{dx}\arccos(3x^3)$.

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7. $\dfrac{d}{dy}e^y \ln y$.

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8. $\dfrac{d}{dx}\exp\left(4/x^3\right)$.

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9. $\dfrac{d}{dx}\sin\left(\sqrt{1-e^x}\right)$.

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10. $\dfrac{d}{dt}\ln\left(\dfrac{\sqrt{t}}{t}\right)$.

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11. Use logarithmic differentiation to find y' when $y = \dfrac{e^x\sqrt{x}}{x^2+1}$.

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12. Use logarithmic differentiation to find y' when $y = x^{\pi}\pi^{x}$.

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13. Differentiate $y = \ln\left(\dfrac{x^2(x+1)(x-3)}{x+4}\right)$.

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14. Differentiate $y = \log_5(5x^2)$.

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15. Differentiate $y = 4^{2x+3}$.

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16. Solve the differential equation $\frac{dr}{ds} = \frac{3r}{4}$.

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17. Solve the differential equation $\sqrt{x} + \sqrt{y}y' = 0$.

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18. Find the particular solution of the differential equation $\sqrt{x} + \sqrt{y}y' = 0$ that satisfies the initial condition $y(1) = 9$.

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19. Translate this sentence into a differential equation. *The rate of change of the velocity v with respect to time t is directly proportional to the velocity squared.*

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20. Solve the differential equation in the last problem.

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21. Integrate $\int u \sin(u^2)\, du$.

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22. Integrate $\int_0^2 \frac{2x}{\sqrt{5+x^2}}\, dx$.

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23. Integrate $\int \frac{\cos(\ln x)}{x}\, dx$.

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24. Integrate $\int (\sin\theta - \cos\theta)^3 (\cos\theta + \sin\theta)\, d\theta$.

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25. Integrate $\int_0^{\pi/4} \frac{\sin\theta}{\cos^4\theta}\, d\theta$.

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26. Compute $\arctan\left(\frac{\sqrt{3}}{3}\right)$.

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27. Compute $\operatorname{arcsec}(-2)$.

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28. Compute $\cos(\arctan(\sqrt{3}))$.

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29. Integrate $\displaystyle\int_0^2 \frac{dx}{\sqrt{4-x^2}}$. Simplify your answer.

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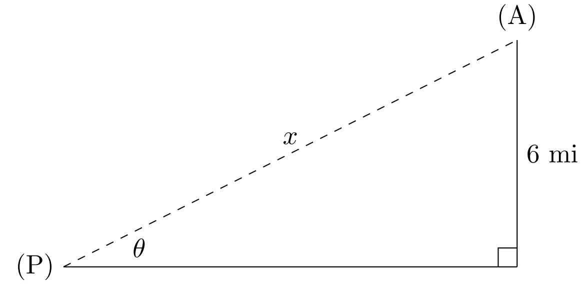
30. An airplane (A) flies at an altitude of 6 miles toward a person standing on the ground (P). If the distance from the airplane to the person is x, find a formula for the angle θ in the figure below.



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31. Integrate $\int 5^{2x}\,dx$ by using the fact that $5^{2x} = e^{2x\ln 5}$.

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32. Integrate $\int 5^{2x}\,dx$ by using the fact that $5^{2x} = (5^2)^x = 25^x$.

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33. $\dfrac{d}{dx}\ln(\ln x)$.

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34. Use logarithmic differentiation to find the derivative of $y = x^{-\ln x}$.

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35. Use your answer to the last problem to find the x-value where the function $y = x^{-\ln x}$ has a maximum. Graph the function to check your answer.

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36. Suppose that a certain population P is growing according to the differential equation $\dfrac{dP}{dt} = \frac{1}{4}P(2-P)$. Make a slope field for this differential equation and use it to describe in words what will happen to the population in the long run if $P(0) = 0.5$.

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37. Use Euler's method with 5 steps and $\Delta x = 0.4$ to approximate the solution of the differential equation $y' = \frac{1}{2}x(3-y)$ if $y(0) = 1$. Complete the following table of values (*accurate to two decimal places*):

x	0.0	0.4	0.8	1.2	1.6	2.0
y	1.0					

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38. Copy or print out and attach your computer code for the last problem.

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39. Initially the temperature of an object is 60°C. The temperature of the object is changing at the rate given by the differential equation $\dfrac{dy}{dt} = -\dfrac{1}{2}(y-20)$ where y is temperature in °C and time t is measured in hours. Use Euler's method with 100 steps to estimate the temperature after 1 hour.

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40. Copy or print out and attach your computer code for the last problem.
