Calculus II - Math 142 **Final Exam**

Name: ______________________

You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of another page and indicate clearly which work goes with which problem.

Problem	Maximum Points	Your Score
1	6	
2	6	
3	8	
4	4	
5	4	
6	10	
7	6	
8	15	
9	6	
10	4	
11	4	
12	6	
13	6	
14	9	
15	6	
Total:	100	

1. (6 points) Find all values of x for which the Taylor series $\sum\limits_{n=0} \frac{2^n}{n} x^n$ converges.

2. (6 points) Solve the differential equation $\frac{dy}{dx} = 3x^2 y$.

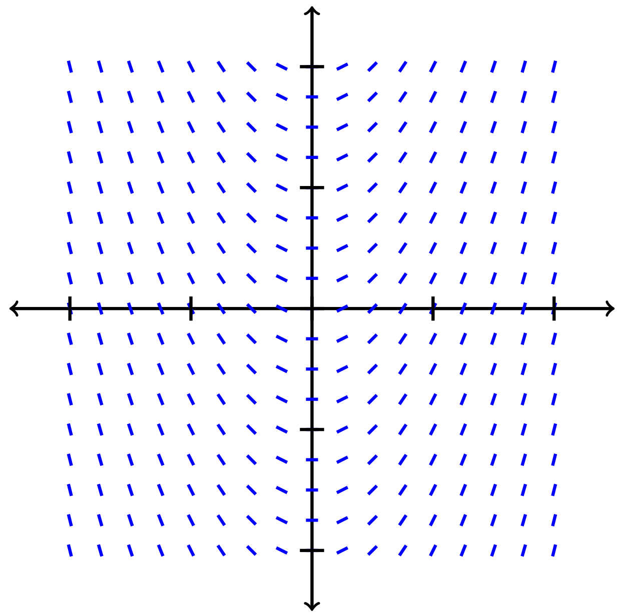
3. (8 points) Use the Maclaurin series for $\sin(x)$ to find Maclaurin series for the following functions.

(a) $\sin(x^3)$

(b) $\displaystyle\int \sin(x^3)\,dx$

4. (4 points) For what values of x does the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x+5)^n}{8^n}$ converge?

5. (4 points) The slope field below corresponds to which of the following differential equations? Explain your answer.



$$(a)\ \frac{dy}{dx} = e^x \qquad (b)\ \frac{dy}{dx} = x^2 + 1 \qquad (c)\ \frac{dy}{dx} = 2x$$

6. (10 points) Find the following integrals.

(a) $\displaystyle\int \cos x\, e^{\sin x}\, dx$

(b) $\displaystyle\int \frac{\cos x}{(\sin x)^2}\, dx$

7. (6 points) Let $\mathcal{R}$ be the region between the curve $y = 4x^2 + 2$, and the lines $x = 1$ and $y = 0$. Find the volume of the solid obtained by revolving $\mathcal{R}$ around the y-axis.

8. (15 points) Find the following integrals.

(a) $\displaystyle\int x^3 e^x \, dx$

(b) $\displaystyle\int \frac{x^2 + 2x + 6}{x + 1} \, dx$

(c) $\displaystyle\int \frac{4}{(x-3)(x+1)} \, dx$

9. (6 points) Complete the following.

   (a) Find the derivative

$$\frac{d}{dx} \ln\left(\frac{\sqrt[3]{x+4}}{x^5(x+1)^3}\right).$$

   (b) Find $\log_2(\frac{9}{16}) + \log_2(\frac{4}{9})$.

10. (4 points) A population of bacteria is growing exponentially according to the equation $P(t) = P_0e^{kt}$. If the initial population was 1000 cells, and the population after 2 hours was 3000 cells, then what is k?

11. (4 points) How many terms of the alternating series

$$1 - 1/4 + 1/9 - 1/16 + 1/25 - 1/36 + \ldots + \frac{(-1)^n}{(n+1)^2} + \ldots$$

would you need to add up in order to approximate the infinite sum with an error **less than** $\frac{1}{100}$?

12. (6 points) Find the following limits. Be sure to show your work.

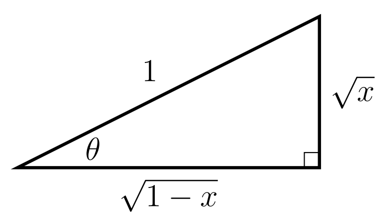
(a) $\displaystyle\lim_{n\to\infty} \frac{\ln n}{n^2}$.

(b) $\displaystyle\lim_{x\to 1} \frac{x^7 - 1}{x^4 - 1}$.

(c) $\displaystyle\lim_{x\to\infty} xe^{-x}$.

13. (6 points) Use the trigonometric substitution $x = \sin^2\theta$ to find $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx$.

(Hint: Be sure to find dx and all of the important trig ratios.)



14. (9 points) i. Determine whether each series below converges or diverges.
ii. Explain what test you used for each.
iii. If the series converges, then find the infinite sum if you can.

(a) $100 + 20 + 4 + \frac{4}{5} + \frac{4}{25} + \ldots$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$.

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{\ln n}$.

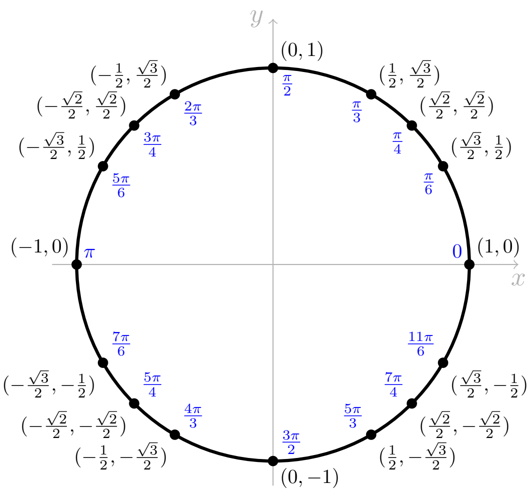
15. (6 points) Find the area between the curves $f(x) = 2x - x^2$ and $g(x) = x$

Formula Sheet

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Common Angles



Selected Derivatives

- $\frac{d}{dx} a^x = a^x \ln a$
- $\frac{d}{dx} \tan x = \sec^2 x$
- $\frac{d}{dx} \sec x = \sec x \tan x$
- $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$
- $\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$
- $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2 - 1}}$

Selected Integrals

- $\int a^u \, du = \frac{a^u}{\ln a} + C$
- $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
- $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$
- $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$
- $\int \sec u \, du = \ln|\sec u + \tan u| + C$

Arc Length

$$L = \int \sqrt{1 + (y')^2} \, dx$$

Volumes of Revolution

$$V = \int \pi R^2 - \pi r^2 \, dx \text{ (or } dy) \qquad V = \int 2\pi r h \, dr$$

Surface Area of Revolution

$$S = \int 2\pi r \sqrt{1 + (y')^2} \, dx$$

Half-Angle Formulas

- $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$
- $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

Alternating Series Error

$$|S - S_n| \leq a_{n+1}$$

Ratio Test

$$\lim_{n \to \infty} \frac{|a_{n+1}|}{|a_n|} > 1 \text{ or } < 1?$$

Taylor Polynomial of Degree n

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x - c)^k$$

Taylor Polynomial Remainder

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x - c)^{n+1}$$

Important Maclaurin Series

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
- $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
- $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$
- $\ln(1 + x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$
- $\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n$