Counting Functions

Permutations

$$ {}_nP_k = (n)_k = \frac{n!}{(n-k)!} $$

Counts: The number of ways to select k objects from a set of n distinct elements <u>without</u> replacement if order <u>matters</u>.

Combinations

$$ {}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!} $$

Counts: The number of ways to select k objects from a set of n distinct elements <u>without</u> replacement if order <u>doesn't</u> matter.

This is also the **binomial coefficient** which is the coefficient of $x^k y^{n-k}$ in the algebraic expansion of $(x+y)^n$.

Strings

$$ n^k $$

Counts: The number of ways to select k objects from a set of n distinct elements <u>with</u> replacement if order <u>matters</u>.

Multicombinations

$$ \left(\!\!\binom{n}{k}\!\!\right) = \binom{n+k-1}{k} $$

Counts: The number of ways to select k objects from a set of n distinct elements <u>with</u> replacement if order <u>doesn't</u> matter.

Distinguished Permutations

$$ \binom{n}{n_1, n_2, \ldots, n_k} = \frac{n!}{n_1! n_2! \cdots n_k!} $$

Counts: The number of different possible orderings of a multiset with k distinct elements that have multiplicities $n_1, \ldots, n_k$.

A **multiset** is like a set except elements can repeat, for example $\{a, a, a, b, b\}$ is a multiset where a has multiplicity 3 and b has multiplicity 2.

This is also the **multinomial coefficient** which is the coefficient of $x_1^{n_1} x_2^{n_2} \cdots x_k^{n_k}$ in the algebraic expansion of $(x_1 + \ldots + x_k)^n$.

Fundamental Counting Principle

Multiplication Rule.

If you have sets $A_1, \ldots, A_k$ with $n_1, \ldots, n_k$ elements respectively, then there are $n_1 \cdots n_k$ ways to choose one element from each set.

Addition Rule.

If you have disjoint sets $A_1, \ldots, A_k$ with $n_1, \ldots, n_k$ elements respectively, then there are $n_1 + \ldots + n_k$ ways to choose one element from the union.