Discrete Probability Distributions

Binomial

$$X \sim \text{Binom}(n, p)$$

Situation: X is the number of successes in n independent Bernoulli trials, which each have probability of success p.

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$
$$E(X) = np$$
$$\text{Var}(X) = np(1-p)$$

Poisson

$$X \sim \text{Pois}(\lambda)$$

Situation: Events are occuring at a fixed rate λ. X is the total number of events that occur. Similar to binomial, except you don't know n, but it is *very* large and you know that $np = \lambda$.

$$P(X = k) = e^{-\lambda} \lambda^k / k!$$
$$E(X) = \lambda$$
$$\text{Var}(X) = \lambda$$

Negative Binomial

$$X \sim \text{Nbinom}(r, p)$$

Situation: How many trials until you get r successes?

$$P(X = n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}$$
$$E(X) = r/p$$
$$\text{Var}(X) = r(1-p)/p^2$$

Hypergeometric

$$X \sim \text{Hyper}(N, M, n)$$

Situation: A population of size N contains M successes. X is the number of successes in n trials *without replacement*.

$$P(X = k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$$
$$E(X) = np$$
$$\text{Var}(X) = np(1-p)\left(1 - \tfrac{n-1}{N-1}\right)$$

where $p = M/N$.

Geometric

$$X \sim \text{Geom}(p)$$

Situation: How many trials until you get one success?

$$P(X = n) = (1-p)^{n-1} p$$
$$E(X) = 1/p$$
$$\text{Var}(X) = (1-p)/p^2$$