Rules of Algebra

For adding and multiplying real numbers.

Addition Rules

1. **Associative** $a + (b + c) = (a + b) + c$
2. **Commutative** $a + b = b + a$
3. 0 is the **Additive Identity**.
4. **Additive Inverses** are negatives:

$$a + (-a) = 0.$$

Multiplication Rules

1. **Associative** $a(bc) = (ab)c$
2. **Commutative** $ab = ba$
3. 1 is the **Multiplicative Identity**.
4. **Multiplicative Inverses** are repicricals:

$$a\left(\frac{1}{a}\right) = 1.$$

Distributive Law

$$a(b + c) = ab + ac$$

Notes

- There are no extra rules for subtraction and division.
- Subtraction is just addition by negatives.
- Division is just multiplication by recipricals.
- You don't need to memorize the names associative and commutative. Just know that if you have several terms added together, then neither the order of the terms nor the order you try to add them matters. Same with several factors multiplied together.
- The distributive law controls everything about how addition and multiplication mix together.

Factors vs. Terms

Terms

Numbers and expressions being added/subtracted.

Factors

Numbers and expressions being multiplied/divided.

Notes

- Distribution expands factors into terms.

$$a(b+c) = ab + ac \qquad (x+2)(x+3) = x^2 + 5x + 6$$

- Factoring un-distributes terms back into factors.

$$a(b+c) = ab + ac \qquad (x+2)(x+3) = x^2 + 5x + 6$$

- Factors cancel in fractions, but terms don't.

$$\frac{\cancel{(x-3)}(x+5)}{(x+1)\cancel{(x-3)}} = \frac{(x+5)}{(x+1)} \quad \text{and} \quad \frac{\cancel{4}\cancel{x}y}{\cancel{12}x^{\cancel{2}}} = \frac{y}{3x} \quad \text{but} \quad \underbrace{\frac{4x+3}{4x+7} \neq \frac{3}{7}}_{\text{can't cancel the } 4x \text{ term}} \quad \text{and} \quad \underbrace{\frac{4x+3}{4x+7} \neq \frac{x+3}{x+7}}_{\text{can't cancel the 4 either}}$$

- Powers distribute to factors, but not to terms.

$$(ab)^3 = a^3b^3 \quad \text{and} \quad \sqrt{9x^2} = 3x \quad \text{but} \quad (a+b)^3 \neq a^3 + b^3 \quad \text{and} \quad \sqrt{9+x^2} \neq 3 + x$$

Exponent Rules

Powers represent repeated multiplication

$$a^m = \underbrace{a \cdot a \cdot a \cdots a}_{m\text{-copies}}.$$

Therefore these rules are true:

1. $a^0 = 1$
2. $(a^m)(a^n) = a^{m+n}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^n = a^n b^n$
6. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Negative Powers

Negative powers are reciprocals.

$$a^{-n} = \frac{1}{a^n}$$

Fractional Powers

Fractional powers include square & cube roots.

$$\sqrt[n]{a} = a^{1/n}$$

Notes

- Every exponent rule corresponds to a fact about addition & multiplication if you just change the powers to products and the multiplication/division to addition/subtraction. For example:

$\underbrace{a^0 = 1}_{\text{1 is the multiplicative identity}}$	is just like	$\underbrace{0a = 0}_{\text{0 is the additive identity}}$
$\underbrace{(a^m)(a^n) = a^{m+n}}_{(m+n)\text{-copies of } a \text{ multiplied together}}$	corresponds to	$\underbrace{ma + na = (m+n)a}_{(m+n)\text{-copies of } a \text{ added together}}$
$\underbrace{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}_{n\text{-copies of } \frac{a}{b} \text{ multiplied together}}$	corresponds to	$\underbrace{n(a-b) = na - nb}_{n\text{-copies of } (a-b) \text{ added together}}$

- Negative powers don't mean the result is negative. And fractional powers don't mean the result is a fraction. Don't confuse powers with multiplication!
- Simplify rational powers by splitting them up: $8^{2/3} = (8^{1/3})^2 = (2)^2 = 4$.