Infinite Series Summary **Math 142**

Geometric Series are sums where each term is exactly r times the term before it, where r is always the same number (called the **common ratio**). If $|r| < 1$, then the series will converge and the sum of the series is:

$$a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}. \qquad \text{(Geometric Sum)}$$

p-Series are series of the form:

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots.$$

Unlike geometric series, these don't have a nice formula for the sum, but it is important to know when they converge. A p-series converges if $p > 1$, and it diverges otherwise. When $p = 1$, you get the harmonic series which is a simple example of a series that diverges even though the terms approach zero.

Alternating Series have alternating positive and negative terms. Any alternating series can be written as:

$$\sum_{n=0}^{\infty} (-1)^n b_n = b_0 - b_1 + b_2 - b_3 + b_4 - \ldots$$

As long as the terms b_n are (1) getting smaller, and (2) approaching zero, then the series will converge. One nice thing about alternating series is that you can estimate how far off a partial sum S_n is from the infinite sum S using the next term in the series:

$$\text{Error} = |S - S_n| \le b_{n+1}. \qquad \text{(Alt. Series Error)}$$

Power Series have a variable x being raised to a power in each term:

$$\sum_{n=0}^{\infty} a_n (x-c)^n.$$

The constant c is the **center** of the power series. Every power series has an **interval of convergence** around $x = c$ that has a radius of

$$R = \lim_{n\to\infty} \frac{|a_n|}{|a_{n+1}|}. \qquad \text{(Radius of Conv.)}$$

Inside the radius of convergence, the power series will converge to some function $f(x)$, and that function determines the coefficients a_n according to the Taylor series formula:

$$a_n = \frac{f^{(n)}(c)}{n!}. \qquad \text{(Taylor Coeff.)}$$

Important examples include:

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$
- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} - \ldots$
- $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} - \ldots$
- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$

There are several ways to make new power series from old ones, including:

- Multiplying
- Dividing
- Differentiating
- Integrating
- Substituting for x.