Power Series Math 142

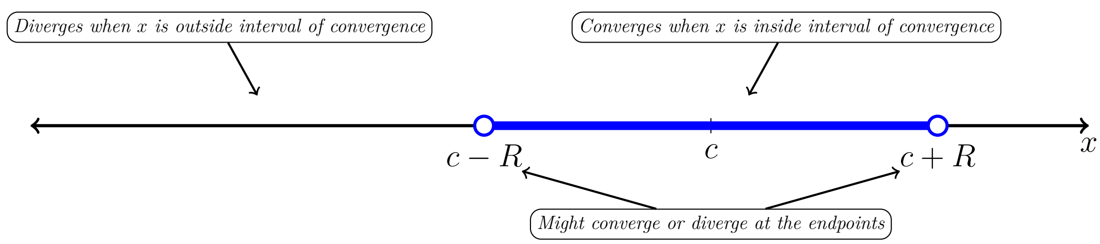
A **power series** is a series of the form

$$\sum_{n=0}^{\infty} a_n (x-c)^n$$

where the coefficients a_n don't depend on x. The number c is called the **center** of the series. Every power series has a **radius of convergence**

$$R = \lim_{n\to\infty} \frac{|a_n|}{|a_{n+1}|}.$$

The center and radius of convergence determine an **interval of convergence** from $c-R$ to $c+R$:



When a power series converges, the value of the sum is a function $f(x)$. The coefficients a_n determine the derivatives of f, and vice versa! In fact:

$$\sum_{n=0}^{\infty} a_n (x-c)^n = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n,$$

or put another way, the coefficients are $a_n = \frac{f^{(n)}(c)}{n!}$ where $f^{(n)}(c)$ is the n^{th} derivative of f evaluated at $x = c$. When a series is derived using the formula on the right it is called a **Taylor series** but that is really just another name for a power series. If the center is $c = 0$, then we sometimes call the series a **Maclaurin series**. Here are the most important Maclaurin series.

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$
- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$
- $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$
- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$

Exercise. Find the radius and interval of convergence for each of these power series.