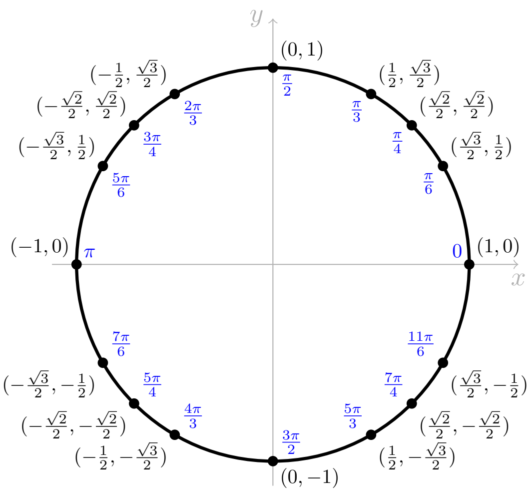
Formula Sheet

Quadratic Formula

$$ax^2 + bx + c = 0 \text{ when } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Common Angles



Secondary Trigonometry Ratios

$$\sec x = \frac{1}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \csc x = \frac{1}{\sin x}$$

Trig Product Formulas

$$\begin{aligned}
\cos(\alpha)\cos(\beta) &= \tfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)] \\
\cos(\alpha)\sin(\beta) &= \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\beta-\alpha)] \\
\sin(\alpha)\cos(\beta) &= \tfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] \\
\sin(\alpha)\sin(\beta) &= \tfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]
\end{aligned}$$

Change of Base Formulas

$$a^x = e^{x \ln a} \qquad \log_a(x) = \frac{\ln x}{\ln a} \qquad \text{(both for any } a > 0)$$

Selected Derivatives

$$\tfrac{d}{dx} a^u = (a^u \ln a)u' \qquad \tfrac{d}{dx} \log_a(u) = \frac{u'}{u \ln a}$$

$$\tfrac{d}{dx} \tan u = (\sec^2 u)u' \qquad \tfrac{d}{dx} \sec u = (\sec u \tan u)u'$$

$$\tfrac{d}{dx} \cot u = -(\csc^2 u)u' \qquad \tfrac{d}{dx} \csc u = -(\csc u \cot u)u'$$

$$\tfrac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \tfrac{d}{dx} \arccos u = \frac{-u'}{\sqrt{1-u^2}}$$

$$\tfrac{d}{dx} \arctan u = \frac{u'}{1+u^2} \qquad \tfrac{d}{dx} \operatorname{arccot} u = \frac{-u'}{1+u^2}$$

$$\tfrac{d}{dx} \operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx} \operatorname{arccsc} u = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Selected Integrals

$$\int a^u \, du = \frac{a^u}{\ln a} + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$$

Riemann Sum

$$A \approx \sum_{k=1}^{n} f(x_k)\Delta x, \quad \Delta x = \frac{b-a}{n}, \quad x_k = a + k\Delta x$$

Euler's Method

$$y_{n+1} = y_n + F(x_n, y_n)\Delta x, \qquad x_n = x_0 + n\Delta x$$

Arc Length

$$L = \int \sqrt{1 + (y')^2} \, dx \text{ or } L = \int \sqrt{(\tfrac{dx}{dt})^2 + (\tfrac{dy}{dt})^2} \, dt$$

Volumes of Revolution

$$V = \int \pi R^2 - \pi r^2 \, dx \text{ (or } dy), \quad V = \int 2\pi r h \, dr$$

Average Values of a Region

$$\bar{x} = \frac{1}{A} \int_a^b x[f(x) - g(x)] \, dx, \quad \bar{y} = \frac{1}{2A} \int_a^b [f^2(x) - g^2(x)] \, dx$$

Work

$$W = \int F \, dx \quad \textit{or} \quad W = \int x \, dF$$

Radius of Convergence (for $\sum_{n=1}^{\infty} a_n (x-c)^n$)

$$R = \lim_{n \to \infty} \frac{|a_n|}{|a_{n+1}|}$$

Taylor Polynomial and Remainder

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1} \text{ (} z \text{ is between } x \text{ \& } c)$$

Important Maclaurin Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \qquad R = \infty$$

$$\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \qquad R = 1$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} \qquad R = 1$$