Math 105 - Homework 5

Name:

Solve the following without using a calculator.

- 1. If f(x) = 5 + x and $g(x) = \sqrt{x}$, then what are f(g(4)) and g(f(4))?
- 2. Suppose $f(x) = x^2 4$ and g(x) = 2 3x. How far apart are f(3) and g(3) on a number line?
- 3. The function $f(x) = \frac{1}{2} \left(x + \frac{5}{x}\right)$ can be used to calculate the square root of 5. Find f(5) and f(f(5)). Cool fact: if you kept going, every extra time you apply the function f to the previous answer, you would get closer and closer to $\sqrt{5}$ which is approximately 2.236068.
- 4. Suppose that the population of a certain species is represented by the variable x. Let f(x) be a function that predicts what the population will be one year later, based on the current population. What would the function f(f(x)) represent?
- 5. The following graphs show two different functions f(x) and g(x).



Use the graphs to evaluate g(f(4)) and f(g(1)).

6. Sketch a graph of the function $f(x) = 4 - (x+1)^2$ by plotting the y-values at $x = 0, \pm 1$, and ± 2 and then filling in the rest of the graph.

7. The amount of garbage produced by a city (measured in tons per week) is given by a function g(p) where the variable p is the city's population measured in thousands of people. One city has a population of 40,000 people and produces 13 tons of garbage each week. In function notation, this would be expressed as (fill in the blanks):

$$g() =$$

- 8. The inverse of the function g in the last problem would be written g^{-1} . Explain what the information $g^{-1}(5) = 18$ would tell us about a city. That is, what is its population and garbage production?
- 9. Suppose that f(x) is a linear function such that 3 = f(0) and 5 = f(1). Find the formula for f(x).

10. If f(x) = 3x + 1, then what is f(f(x))? Simplify your answer.

- 11. The time in seconds that it takes a pendulum to complete a full oscillation (swing back and forth) is $T = 2\pi \sqrt{\frac{L}{9.8}}$ where L is the length of the pendulum in meters. Find the inverse of this function.
- 12. The function $A(r) = \pi r^2$ computes the area of a circle of radius r. Find the formula for the inverse function and describe in words what it computes about a circle.
- 13. What is the domain of the function $h(x) = \sqrt{6-x}$? That is, what x-values make sense as inputs?

14. Use the graph below to find the values of x for which $f(x) = x^3 - 13x + 12 > 0$.



- 15. A bakery sells cupcakes. If they gave away cupcakes for free, people would demand 1200 cupcakes per day. For every dollar the price of a cupcake increases above 0, they will sell 200 fewer cupcakes per day. Find a formula for the quantity of cupcakes Q(p) they will sell as a function of the price p of a cupcake in dollars.
- 16. Find the total revenue R(p) that the bakery in the previous problem will earn selling cupcakes as a function of p. Recall that revenue is price times quantity sold.

A store can produce souvenir T-shirts at a cost of \$2 each. They need to choose a price for the shirts. If they sell the shirts for \$5 each, they will sell 4,000 shirts. If they raise the price, then for each \$1 increase in price, 400 fewer shirts will be sold. Using the variable p to represent the price that the store charges, find each of the following functions:

17. Quantity of shirts sold: Q(p)

- 18. Revenue (total money they get from selling shirts): R(p)
- 19. Cost (total money they spend to make the shirts): C(p)
- 20. Profit (revenue minus cost): P(p)