Homework 10 - Computer Science 461 **Name:** ______________________

Due Monday, November 13.

1. Recall that a graph G is **bipartite** if it is 2-colorable, i.e., the vertices of G can be colored either white or black so that every edge connects one black vertex with one white vertex. Let

$$\text{BIPARTITE} = \{\langle G \rangle : G \text{ is a bipartite graph}\}.$$

Prove that BIPARTITE is in class NP by describing what counts as a **solution** for a graph G and describing a **verifier** algorithm that can confirm the solution is correct in polynomial time.

2. A graph is **connected** if there is a path from any vertex to any other. Let

$$\text{CONNECTED} = \{\langle G \rangle : G \text{ is a connected graph}\}.$$

Consider the following algorithm to decide whether a graph is connected:

- **Step 1.** Select the first vertex of G and mark it.
- **Step 2.** Loop through the edges of G. For any edge that touches one marked vertex, mark the other vertex it touches.
- **Step 3.** Repeat step 2 until you don't find any more vertices to mark.
- **Step 4.** Loop through all of the vertices and check that they are marked. Reject if any are not marked, otherwise accept.

Determine the run time of the algorithm in big-O notation as a function of the number of vertices (n) in the graph. Hint: What is the maximum number of edges a graph with n vertices can have?

3. The Kleene star of a language is the set $L^* = \{w_1 w_2 \dots w_k : k \in \mathbb{N} \text{ and each } w_i \in L\}$. Prove that if L is a language in class P, then so is L^*. Hint: Use a dynamic programming algorithm that stores a table of substrings of w and whether they are in L^*, ordered by length. You'll need to describe how the algorithm works to decide L^*.

4. Given two strings $a, b \in \{0,1\}^*$, can we decide if a and b are equal in polynomial time? Explain why or why not.