Homework 11 - Computer Science 461 **Name:** ______________________

Due Monday, November 20.

1. Recall that the language PATH is in class P:

$$\text{PATH} = \{\langle G, s, t\rangle : G \text{ is a graph with a path from vertex } s \text{ to vertex } t\}.$$

   It is not known whether PATH is NP-complete, however most computer scientists believe that it is not. Prove that if PATH is NP-complete, then P = NP.

2. The subset-sum problem asks whether any subset of the entries in an integer array adds up to a given sum. It corresponds to the following language:

$$\text{SUBSET-SUM} = \{\langle A, t\rangle : A \text{ is an array of integers with a subset that adds up to } t\}.$$

   For example $\langle [3, 5, 10, 10], 23\rangle \in$ SUBSET-SUM because $3 + 10 + 10 = 23$, but there is no subset of the entries of $[3, 5, 10, 10]$ that adds up to 4, so $\langle [3, 5, 10, 10], 4\rangle \notin$ SUBSET-SUM. Prove that SUBSET-SUM is in class NP.

3. The partition problem asks whether it is possible to separate the elements of an integer array into two disjoint subsets so that the sums of both subsets are equal. All of the elements in the array must be in exactly one of the subsets (so the subsets are a partition of the array). This problem is equivalent to deciding

$$\text{PARTITION} = \{\langle A \rangle : A \text{ is an array of integers partitionable into 2 subsets with equal sums}\}.$$

Prove that there is a polynomial-time reduction from PARTITION to SUBSET-SUM, in other words, PARTITION $\leq_P$ SUBSET-SUM.

4. Let L be a language in class NP. Prove that L^* is also in class NP by describing a nondeterministic algorithm that can decide L^* in polynomial time.