Homework 6 - Computer Science 461   **Name:** _____
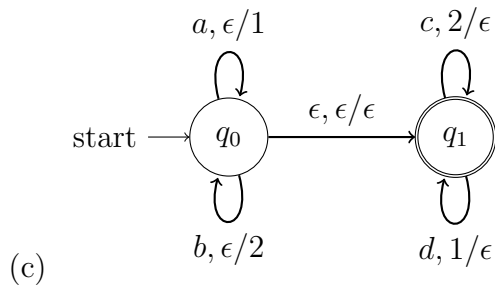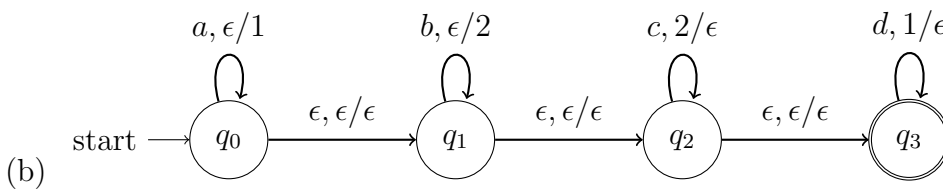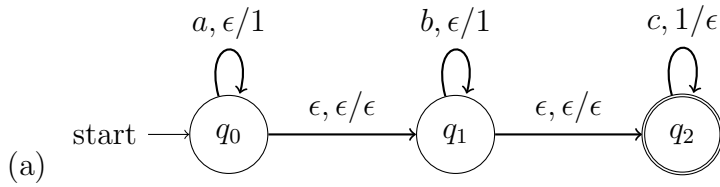
Due Monday, October 9.

1. Identify the context-free language that is accepted by each of the following nondeterministic pushdown automata (NPDA). Explain (briefly) you answer for each.

(a)

$$a, \epsilon/1 \qquad b, \epsilon/1 \qquad c, 1/\epsilon$$

start $\longrightarrow$ q_0 $\xrightarrow{\epsilon, \epsilon/\epsilon}$ q_1 $\xrightarrow{\epsilon, \epsilon/\epsilon}$ q_2

(b)

$$a, \epsilon/1 \qquad b, \epsilon/2 \qquad c, 2/\epsilon \qquad d, 1/\epsilon$$

start $\longrightarrow$ q_0 $\xrightarrow{\epsilon, \epsilon/\epsilon}$ q_1 $\xrightarrow{\epsilon, \epsilon/\epsilon}$ q_2 $\xrightarrow{\epsilon, \epsilon/\epsilon}$ q_3

(c)

$$a, \epsilon/1 \qquad c, 2/\epsilon$$

start $\longrightarrow$ q_0 $\xrightarrow{\epsilon, \epsilon/\epsilon}$ q_1

$$b, \epsilon/2 \qquad d, 1/\epsilon$$

2. Let $\Sigma = \{(,), [,]\}$. That is, Σ is the alphabet consisting of the four symbols (,), [, and]. Let L be the language over Σ consisting of strings in which both parentheses and brackets are balanced. For example, the string ([][()()])([]) is in L but [()) is not. Find a NPDA that accepts this language. Hint: *you should only need one state!*

3. Use the pumping lemma to prove that the language $\{a^n b^n a^n b^n : n \in \mathbb{N}\}$ is not context-free.

4. Is the language $\{a^m + a^n = a^{m+n} : m, n \in \mathbb{N}\}$ over the alphabet $\Sigma = \{a, +, =\}$ context-free? Explain why or why not.

5. For any languages A and B, let $A \diamond B = \{xy : x \in A, y \in B, \text{ and } |x| = |y|\}$. If A and B are regular languages, prove that $A \diamond B$ is a context-free language by describing how you could use NFAs for A and B to construct a NPDA for $A \diamond B$.