**Homework 7 - Computer Science 461** **Name:** ______________________

*Due Wednesday, October 18.*

1. Give a detailed written description (but not a state diagram) of a Turing machine that accepts the following languages.
   (a) $L = \{w \in \{a,b\}^* : w$ has an equal number of $a$'s and $b$'s$\}$.

   (b) $L = \{a^n b^{2n} c^{3n} : n \geq 1\}$.

2. Let $\Sigma = \{a\}$. Draw a state diagram for a Turing machine that evaluates the function $f : \Sigma^* \to \Sigma^*$ defined by $f(a^n) = a^{2n}$.

3. A `binary-incrementer` is a function that reads a binary number from a tape, and replaces it with the binary number that is one greater. So 111 becomes 1000, for example. Draw a state diagram for a Turing machine that evaluates the `binary-incrementer` function.

4. If you have a Turing machine that computes the `binary-incrementer` function, explain how you could create a Turing machine that reads a string of $n$ 1's, and replaces it with the binary integer that represents $n$. For example 1111 would become 100 since 100 represents $n = 4$ in binary.

5. Let $\Sigma$ be an alphabet, and let $L \subset \Sigma^*$ be a language. If $L$ is decidable, prove that its complement $\overline{L}$ is also decidable.

6. Why doesn't the same argument show that the complement of an acceptable language is acceptable?