# Infinite Series Summary — Math 142

*Know how these four special types of infinite series work.*

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**Geometric Series** converge if and only if the **common ratio** $r$ has $|r| < 1$. Then the sum is:

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}. \qquad \text{(Geometric Sum Formula)}$$

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**p-Series** converge if and only if $p > 1$.

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots.$$

When $p = 1$, you get the harmonic series which is a simple example of a series that diverges even though the terms approach zero.

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**Alternating Series** If the terms $b_n$ are (1) decreasing and (2) approach zero, then it will converge.

$$\sum_{n=0}^{\infty} (-1)^n b_n = b_0 - b_1 + b_2 - b_3 + b_4 - \ldots.$$

For alternating series, you can estimate how far a partial sum $S_n$ is away from the infinite sum $S_\infty$ using the next term in the series:

$$|\text{Error}| = |S_\infty - S_n| \le b_{n+1}. \qquad \text{(Alternating Series Error Formula)}$$

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**Power Series** are functions of a variable $x$ and converge for all $x$ inside an **interval of convergence**.

$$f(x) = \sum_{n=0}^{\infty} a_n (x-c)^n.$$

Use the **ratio test** to find the interval of convergence. The coefficients $a_n$ are:

$$a_n = \frac{f^{(n)}(c)}{n!}. \qquad \text{(Taylor Coefficient Formula)}$$

Important power series examples include:

- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$
- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} - \ldots$
- $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} - \ldots$
- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots$

There are several ways to make new power series from old ones, including:

- Multiplying
- Dividing
- Differentiating
- Integrating
- Substituting for $x$.

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