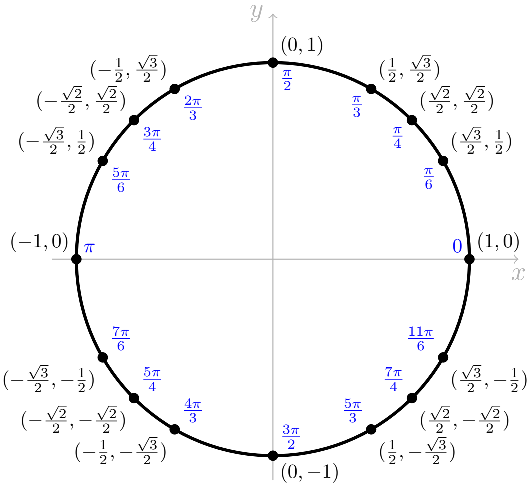
Formula Sheet

Quadratic Formula

$$ax^2 + bx + c = 0 \text{ when } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Common Angles



Secondary Trigonometry Ratios

$$\sec x = \frac{1}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} \qquad \csc x = \frac{1}{\sin x}$$

Double-Angle Formulas

$$\sin 2\theta = 2\cos\theta\sin\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

Half-Angle Formulas

$$\sin^2\theta = \tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta \qquad \cos^2\theta = \tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta$$

Change of Base Formulas

$$a^x = e^{x\ln a} \qquad \log_a(x) = \frac{\ln x}{\ln a} \qquad \text{(both for any } a > 0)$$

Selected Derivatives

$$\tfrac{d}{dx}a^u = (a^u \ln a)u' \qquad \tfrac{d}{dx}\log_a(u) = \frac{u'}{u\ln a}$$

$$\tfrac{d}{dx}\tan u = (\sec^2 u)u' \qquad \tfrac{d}{dx}\sec u = (\sec u\tan u)u'$$

$$\tfrac{d}{dx}\cot u = -(\csc^2 u)u' \qquad \tfrac{d}{dx}\csc u = -(\csc u\cot u)u'$$

$$\tfrac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}} \qquad \tfrac{d}{dx}\arccos u = \frac{-u'}{\sqrt{1-u^2}}$$

$$\tfrac{d}{dx}\arctan u = \frac{u'}{1+u^2} \qquad \tfrac{d}{dx}\operatorname{arccot} u = \frac{-u'}{1+u^2}$$

$$\tfrac{d}{dx}\operatorname{arcsec} u = \frac{u'}{|u|\sqrt{u^2-1}} \qquad \frac{d}{dx}\operatorname{arccsc} u = \frac{-u'}{|u|\sqrt{u^2-1}}$$

Riemann Sum

$$A \approx \sum_{k=1}^{n} f(x_k)\Delta x, \quad \Delta x = \frac{b-a}{n}, \quad x_k = a + k\Delta x$$

Selected Integrals

$$\int a^u\,du = \frac{a^u}{\ln a} + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\,du = -\ln|\csc u + \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$$

Euler's Method

$$y_{n+1} = y_n + F(x_n, y_n)\Delta x, \qquad x_n = x_0 + n\Delta x$$

Arc Length

$$L = \int \sqrt{1+(y')^2}\,dx \text{ or } L = \int \sqrt{(\tfrac{dx}{dt})^2 + (\tfrac{dy}{dt})^2}\,dt$$

Volumes of Revolution

$$V = \int \pi R^2 - \pi r^2\,dx \text{ (or } dy), \quad V = \int 2\pi rh\,dr$$

Work

$$W = \int F\,dx \quad \textit{or} \quad W = \int x\,dF$$

Ratio Test (for $\sum_{n=1}^{\infty} a_n$)

$$\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|}, \quad \begin{cases} < 1 \implies \text{converges absolutely,} \\ = 1 \implies \text{inconclusive,} \\ > 1 \implies \text{diverges.} \end{cases}$$

Taylor Polynomial and Remainder

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1} \text{ (} z \text{ is between } x \text{ \& } c)$$

Important Maclaurin Series

$$e^x = \sum_{n=0}^{\infty} \tfrac{x^n}{n!} \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} \tfrac{(-1)^n x^{2n+1}}{(2n+1)!} \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} \tfrac{(-1)^n x^{2n}}{(2n)!} \qquad R = \infty$$

$$\arctan x = \sum_{n=0}^{\infty} \tfrac{(-1)^n x^{2n+1}}{2n+1} \qquad R = 1$$

$$\ln(1+x) = \sum_{n=0}^{\infty} \tfrac{(-1)^n x^{n+1}}{n+1} \qquad R = 1$$