Homework 10 - Math 421   **Name:** _____

Due Friday, November 10. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. Patty and Selma both work at the DMV. When Patty helps a customer, she always finishes in exactly 10 minutes. When Selma helps a costumer, the amount of time it takes has an Exponential distribution with mean 10 minutes (so the rate is $\lambda = 0.1$ customers per minute). Suppose you walk into the DMV at some random time and see that both Patty and Selma are helping customers. Let X be the amount of time until Patty finishes with her customer and Y be the amount of time until Selma finishes with hers.

   (a) What are the probability distributions for X and Y?

   (b) Find $P(X \leq Y)$.

   (c) Let $Z = \min(X, Y)$. Find the CDF for Z. Hint: Since X and Y are independent,

$$P(X \geq x \text{ and } Y \geq y) = P(X \geq x)P(Y \geq y).$$

2. Suppose that X and Y are random variables with joint probability density function

$$f(x,y) = \begin{cases} 2e^{-2x}/x & 0 \leq x < \infty, \ 0 \leq y \leq x \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $E(X)$ and $E(Y)$. Hint: Use 2D LOTUS.

(b) Find $\text{Cov}(X,Y)$.

3. Roll two 6-sided dice and let F be the value of the first die and T be the total of the two dice. Find $\text{Cov}(F,T)$. Hint: $T = F + S$ where S is the value of the second die.

4. An urn contains 20 balls (10 red and 10 blue). Suppose you draw a sample of 4 balls from the urn. Let R be the number of red balls and B be the number of blue balls. Find the covariance of R and B.