Homework 11 - Math 421 **Name:** _____

Due Friday, November 17. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. Professor Xavier's class has a midterm worth 30% of the final grade and a final exam that is worth the other 70%. Grades on the midterm exam are normally distributed with mean 60 and standard deviation 10. Grades on the final are normally distributed with mean 70 and standard deviation 15.
   (a) Let M be a student's midterm score and F be their final exam score, so their final grade is $G = 0.3M + 0.7F$. Find $E(G)$.

   (b) Suppose that scores on the midterm and final exam are correlated with a correlation coefficient $\rho = 0.6$. Find $\mathrm{Var}(G)$.

   (c) Assuming that G is normally distributed, what percent of students finish Professor Xavier's class with a final grade of 80 or higher?

2. Find the PDF of X^3 for $X \sim \mathrm{Exp}(\lambda)$.

3. Let $U \sim \text{Unif}(-\frac{\pi}{2}, \frac{\pi}{2})$. Find the PDF of $T = \tan(U)$. Be sure to say what the support for T is.

4. Let $U \sim \text{Unif}(0, 1)$ and $X \sim \text{Exp}(1)$ be independent random variables. Use a convolution integral to find the PDF for $U + X$. Hint: To calculate the convolution, you'll need to consider two cases separately: When $t \geq 1$ and when $0 \leq t < 1$.