Homework 2 - Math 421 **Name:** _____

Due Friday, September 1. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. A woman is pregnant with twin boys. Twins may be either identical or fraternal (non- identical). In general, 1/3 of twins born are identical. Obviously, identical twins must be of the same sex; fraternal twins may or may not be. Assume that identical twins are equally likely to be both boys or both girls, while for fraternal twins all possibilities are equally likely. Given the above information, what is the probability that the woman's twins are identical?

2. In tennis, the serving player has two chances to get the ball into play. The first serve is in about 60% of the time, and if it is in, the serving player wins the point about 75% of the time. If the first serve is out, the server gets a second chance to serve the ball. This time, there is an 85% chance the ball will be in, and if it is, the serving player has a 60% chance to win the point. If both serves are out, then the other player automatically wins the point. Make a weighted tree diagram for this situation and use it to answer the following questions.

   (a) Find P(Serving player wins the point).

   (b) Find P(First serve in | Server wins the point).

3. According to the CDC (Centers for Disease Control and Prevention), men who smoke are 23 times more likely to develop lung cancer than men who don't smoke. Also according to the CDC, 21.6% of men in the U.S. smoke. What is the probability that a man in the U.S. is a smoker, given that he develops lung cancer?

4. A family has three children, Alice, Bob, and Carol.

   (a) Find the conditional probability $P(\text{Alice is older than Bob} \mid \text{Alice is older than Carol})$.

   (b) Are the events "Alice is older than Bob" and "Alice is older than Carol" independent? Explain how you can tell.

5. $2n$ balls are chosen at random from a total of $2n$ red balls and $2n$ blue balls. Find a combinatorial expression for the probability that the chosen balls are equally divided in color.

6. **Stirling's formula** says that $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$.

   (a) Use Stirling's formula to show that $\binom{2n}{n} \approx \frac{2^{2n}}{\sqrt{\pi n}}$.

   (b) Use part (a) to find a simple approximation formula for the answer to problem 5.