Homework 3 - Math 421 **Name:** _____

Due Friday, September 8. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. Suppose that 36% of families have a pet dog (but no cat), 22% of families have a pet cat (but no dog), and 8% have both a pet cat and a dog.
   (a) What percent of families don't have a cat or a dog?

   (b) What are the *odds* that a family has a pet cat?

2. If you find out that a family has a pet dog, then what are the posterior odds that they also have a cat? Also, what is the likelihood ratio for having a pet cat for families with dogs versus those without?

3. If you roll two six-sided dice, what is the conditional probability that at least one lands on 6 given that the two numbers are different?

4. A class has 6 freshman boys, 9 freshman girls, and 8 sophomore boys. How many sophomore girls would have to be in the class in order for gender and class to be independent?

5. A *flush* in poker is when 5 cards all have the same suit (hearts, diamonds, clubs, or spades).
   (a) Find the probability that a random 5-card hand from a deck of 52 playing cards is a flush.

   (b) Find the conditional probability that a random 5-card hand is a straight flush given that it is a flush. A *straight flush* is when the cards all have the same suit and form an unbroken sequence of ranks (for example, 7-8-9-10-Jack). The ranks are ordered 2 (low), 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, ace (high).

6. Let A, B, and C be events. If $P(A|C) > P(B|C)$ and $P(A|C^c) > P(B|C^c)$, prove that $P(A) > P(B)$. Hint: *Use the Law of Total Probability* (Theorem 2.3.6 in the book).

7. An event B carries negative information about A, we write $B \searrow A$, if $P(A|B) \leq P(A)$. For each of the following statements, prove or give a counterexample:
   (a) If $B \searrow A$, then $A \searrow B$.

   (b) If $A \searrow B$ and $B \searrow C$, then $A \searrow C$.