Homework 4 - Math 421 **Name:** ________________________

Due Friday, September 15. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. Two balls are randomly selected from an urn containing 4 red, 3 blue, and 2 gray balls. Suppose that we win \$2 for each blue ball, but lose \$1 for each red ball, and gray balls don't win or lose anything. Let X denote our total winnings. Find and graph the probability mass function for X. Be sure to clearly label every possible outcome and its probability.

2. Suppose you roll 2 six-sided dice. Let Y be the minimum value of the two dice. Find and graph the probability mass function for Y.

3. There are one hundred boxes. The first box contains \$1, the second \$2, and so on until the last box which contains \$100. Suppose you choose 5 boxes at random (without replacement). Let M be amount of money in the most valuable of the 5 boxes. Find a formula for the probability mass function $P(M = k)$. Your formula should be a simple expression in terms of binomial coefficients. Hint: *If your most valuable box contains k dollars, then how many different possible combinations of the other boxes are possible?*

4. Suppose that $X \sim \text{Bin}(3, 0.5)$ and $Y \sim \text{Bin}(2, 0.8)$. Find $P(X > Y)$.

5. Suppose that I roll a six-side die until it lands on a 6. Let R be the total number of rolls it takes. What is the formula for the PMF of R?

6. Suppose that an airplane can fly as long as at least half of its engines are working. Also assume that the event that one engine fails is independent of whether any other engine fails. Which of the following planes is more likely to have over half of its engines fail: a plane with 4 engines that are each 95% reliable or a plane with just 2 engines that are each 98% reliable?

7. Suppose that $X \sim \text{Bin}(n, p)$. Let k be a fixed constant in $\{0, 1, \dots, n\}$. Find a formula for the value of the parameter $p \in [0, 1]$ that maximizes $P(X = k)$. This value is called the **maximum likelihood estimator** for p. Hint: *Take the derivative of the binomial distribution PMF with respect to p to find the maximum.*