Homework 5 - Math 421   **Name:** _____

Due Friday, September 29. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. Two balls are randomly selected from an urn containing 4 red, 3 blue, and 2 gray balls. Suppose that we win $2 for each blue ball, but lose $1 for each red ball, and gray balls don't win or lose anything. Let X denote our total winnings. Find $E(X)$. *This is the same situation as problem 1 from HW4.*

2. An elementary school has 4 bus routes. Bus 1 has 40 students, bus 2 has 30, bus 3 has 25, and bus 4 has 45 students. If you randomly select one of the students who rides a bus, let X be the number of students who ride their bus. If you randomly select one of the bus drivers, let Y be the number of students who ride their bus. What are $E(X)$ and $E(Y)$? Why does it make sense that $E(X)$ and $E(Y)$ are different?

3. A volunteer needs to collect 5 signatures for a ballot initiative. Assume that each person the volunteer asks has an independent 40% chance to sign the proposal. Let X be the number of people they talk to fill their quota of signatures.

   (a) Find $P(X = 8)$.

   (b) Find $E(X)$.

4. Find $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$ for the random variables X and Y from the problem about school buses.

5. Suppose Z is a random variable with $E(Z) = 1$ and $\mathrm{Var}(Z) = 3$. Find

   (a) $\mathrm{Var}(4 + 3Z)$.

   (b) $E((2 + Z)^2)$. Hint: *Use the fact that* $\mathrm{Var}(X) = E(X^2) - E(X)^2$.

6. An urn has 5 red and 5 blue balls. Suppose you take two balls from the urn. If they have the same color, you win \$3, otherwise you lose \$2. Find the expected value and variance for this game.

7. If $Y \sim \mathrm{Geom}(p)$, prove that $P(Y = n + k | Y > n) = P(Y = k)$.