Homework 6 - Math 421 **Name:** ______________________

Due Friday, October 6. Be sure to show any work you needed to do. You can use a calculator or computer, but give exact (not decimal) answers when possible.

1. A cruise ship is offering a special honeymoon cruise for newlyweds. If there are 1014 newlywed couples on the cruise ship and the cruise is 1-week long, what is the probability that at least one of the couples will have both of their birthdays occur during the 7-day cruise? Assume that everyone's birthdays are independent and equally likely to happen on each of the 52 weeks of the year.

2. A region has three major highways. The number of daily accidents that occur on these highways are Poisson processes with parameters 0.5, 0.8, and 1.2, respectively.
   (a) What is the expected value for the total number of accidents (combined) on all highways in the region today?

   (b) Assuming that the number of accidents that happen on any one highway is independent of the number of accidents on the others, what is the probability that tomorrow there is exactly one accident on each of the three highways?

3. Let X be a Poisson random variable with parameter λ. What value of λ maximizes $P(X = k)$ for a fixed $k \geq 0$?

4. Suppose that $X \sim \text{Pois}(1)$. Find an infinite series for $P(X \text{ is even})$. Then use a computer to sum enough terms to find a reasonably accurate approximation (Desmos will work for this).

5. Use the Maclaurin series for e^x to show that if $X \sim \text{Pois}(1)$, then

$$P(X \text{ is even}) = \tfrac{1}{2}e^{-1}\left(e^{1} + e^{-1}\right).$$

Is this consistent with the approximation you found in the previous problem?

6. Suppose that the lifetime of a light-bulb has probability density function $f(x) = xe^{-x}$, where $x \geq 0$ is the number of years. Find the expected lifetime of that light-bulb.

7. What is the probability that the light-bulb burns out some time during the first year?

8. If $X \sim \text{Unif}(0, 1)$, find $E(X^3)$.

9. If $X \sim \text{Unif}(0, 1)$ and $Y \sim \text{Unif}(0, 2)$ are independent random variables, find $E(X+Y)$ and $\text{Var}(X+Y)$.