Lab 3 - Probability **Math 121**

The Numbers racket was an illegal lottery run by gangs in major cities throughout the United States. One version worked as follows: you choose of the 1000 three digit numbers 000 to 999 and pay your local Numbers runner a dollar to enter your bet. Each day, one three-digit number is chosen at random and pays off $600.

1. What is the expected value (average pay-off) from one Numbers bet?

2. Suppose Molly lives in New York City and plays the numbers game almost every day for several years. Explain what the Law of Large numbers predicts will happen to Molly.

3. How many times would Molly have to win in order to break even if she played 2400 games?

4. If Molly plays the Numbers game 2400 times, the number of times she wins has a binomial probability distribution $B(N, p)$. What is N and what is p?

For the following questions, use the Binomial Distribution Calculator on the course website:

`http://people.hsc.edu/faculty-staff/blins/StatsTools/binomial.html`.

5. According to the binomial distribution calculator
   (a) What is the probability that Molly loses more money than she wins?
   (b) What is the probability that probability that she breaks even exactly?
   (c) What is the probability that she wins more money than she loses?

6. Sketch the probability distribution for the number of times Molly wins as a probability histogram. Show bars for each outcome from 0 to 10 wins.

7. Does it look like the number of times that Molly wins will have an approximately normal probability distribution? Why or why not?

By now it should be clear that some small time Numbers players might come out ahead, and that individual Numbers players have a lot of variability in how well they do. Now we are going to look at Numbers from the perspective of the gangsters who run the game.

8. It is said that the New York City mobster Casper Holstein took as many as 25,000 bets per day in the Prohibition era. That's 150,000 bets per week if he takes Sunday off. How many winning tickets should Casper expect to pay out per week? How much would that cost him? How much can he expect to make in profit each week?

9. The number of tickets Casper has to pay out per week has a binomial distribution with so many bars to either side of the mean that it is approximately normal. What is the mean and standard deviation of this approximately normal distribution (Hint: $\sigma = \sqrt{p(1-p)N}$).

10. Sketch the distribution of the number of tickets that Casper has to pay out per week as an approximately normal distribution. Label the points 0, 1, and 2 standard deviations above and below the mean. For each of those points figure out how much money Casper will make in profit (money in minus money out).

11. Casper would lose money if more than 250 people won the Numbers game in one week. What is the z-value of that event? Could that ever happen? How likely is it?

12. What important concept from probability explains why Casper Holstein's distrubution is approximately normal while Molly's distribution is not? Explain what is going on.