Math 422 - Homework 7 **Due Wednesday, March 28**

1. Before you can determine the half-lives of radioactive isotopes, it is important to know what the background radiation is in a given detector over a period of time. The following dataset contains the numbers of γ-particles that hit a detector during 20 different 10 second intervals.

   `4 3 8 8 7 3 5 5 7 6 7 6 7 6 11 4 6 10 6 5`

   Assuming that the number of γ-particles that hits the detector during each 10 second interval has a Poisson(λ) distribution, find the MLE for λ based on this data.

2. Out of 50 million instant winner lottery tickets, the proportion of winning tickets is p. Suppose that every day, Bob repeatedly buys instant winner ticket until he finds one that is a winner. Here are the number of tickets that Bob bought each day for the last two weeks:

   `2 6 1 14 14 7 8 25 19 5 5 1 1 19`

   By making reasonable assumptions, estimate the MLE for p. Be sure to explain the assumptions you are making.

3. Suppose that $X_1, \ldots, X_n$ are i.i.d. random variables that are uniformly distributed on the interval $[0, \theta]$.

   (a) Show that $2\bar{x}$ is an unbiased estimator for θ.

   (b) It turns out that $\frac{N+1}{N}\max(x_1, \ldots, x_n)$ is also an unbiased estimator for θ. Which of these two unbiased estimators would you say is a better estimator? Why?

4. A 2006 study looked at whether a certain species of spider (*Lycosa ishikariana*) could be found on beaches in Japan versus the average size of a grain of sand (in millimeters) on those beaches. The file: spiders.csv contains data from the study.

   (a) Use the data to find the MLE for the parameters β_0 and β_1 of a logistic regression model for predicting the probability that these spiders are present on a beach where the average sand grain size is x:

   $$\log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x.$$

   (b) Use your values for β_0 and β_1 to estimate the probability that *Lycosa ishikariana* will be found on a beach with sand grains that are 0.6mm on average.