Math 422 - Homework 8 **Due Friday, April 6**

1. Suppose that I three six-sided dice. The first die is normal, the second die is loaded so that it lands on a six 24% of the time, on a one only 12% of the time, and the other four outcomes are all equally likely (16% each). The third die is not loaded, but it only has the numbers 1 through 5, the space where the six should be is blank (counts as zero). Let the parameter θ denote which of the three dice I roll.

   (a) Suppose that I pick a die and roll it. If I get a 3, what is the likelyhood function for θ?

   (b) What if I get a 6? What is the likelyhood function for θ in that case.

   (c) For both of the previous outcomes, compute the posterior distribution based on the uniform prior $\pi(\theta) = \frac{1}{3}$.

   (d) What is the most likely value of θ in each case above?

2. Suppose you toss a coin and put a Uniform($[0.4, 0.6]$) prior on θ, the probability of getting a head on a single toss.

   (a) If you toss the coin n times and obtain n heads, then determine the posterior density of θ.

   (b) Suppose the true value of θ is actually 0.9. Will the posterior distribution ever put any probability mass around $\theta = 0.9$ for any sample results?

   (c) What do you conclude from part (b) about how you should choose your prior?

3. Suppose that we take a random sample from a normally distributed population that has variance $\sigma_0^2 = 1$, but we don't know the mean μ. We assume a normal prior distribution for μ with mean 0 and variance 4. If we take a sample of size 25 and our sample mean is $\bar{x} = 0.7$, then what is the posterior probability that $\mu > 0$?

4. Suppose that $(x_1, \ldots, x_n)$ is a random sample from the uniform distribution on $[0, \theta]$. You don't know the parameter θ, so you start with a Gamma(α,λ) prior distribution for θ.

   (a) What is the likelyhood function for θ given the values of $x_1, \ldots, x_n$?

   (b) What is the posterior distribution for θ?