One-Sample Hypothesis Test for Proportions

Answers a yes/no question about a population proportion p. Lets you decide if there is statistically significant evidence that p is different from a proportion p_0.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

$$H_0\colon\ p = p_0$$
$$H_A\colon\ p \neq p_0$$

- $\hat{p}$ is the sample proportion
- p_0 is the proportion from H_0
- n is the sample size

Steps

1. **Make hypotheses.** H_0 says that there is no difference between the true population proportion p and the number p_0. H_A can be one-sided if you have prior knowledge or two-sided if you aren't sure.
2. **Calculate z-value.** Use the formula.
3. **Find the p-value.** Use the normal distribution on the Probability Distributions app.
4. **Explain what it means.** This works the same as any other hypothesis test. Lower p-values are more significant.

Assumptions

1. **Randomness** You have a simple random sample from a large population (no sample bias).
2. **Normality** Sample size is large enough so that $\hat{p}$ has a roughly normal distribution.

In order to have a big enough sample size, you should expect at least 10 successes and 10 failures in your sample, if the null hypothesis were true. In other words, both p_0N and $(1 - p_0)N$ should be at least 10.

One-Sample Confidence Interval for Proportions

Estimates a population proportion p. We can be confident that the true value of p is between the upper and lower bound from the formula.

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- $\hat{p}$ is the sample proportion
- z^* is the critical z-value
- n is the sample size

Assumptions

These are the same as for the hypothesis test, except that confidence intervals are a little less robust (need even bigger samples). You can make these confidence intervals more robust by using the **plus-4 method**. To make a plus-4 confidence interval, add two fake failures and two fake successes to the data. These work well as long as $n \geq 10$.