**Maximum Likelihood Estimation** **Math 422 - Workshop**

In statistics, we usually assume that the population we are interested in can be described by a probability distribution. For example, we might assume that the heights of students at one college has a normal distribution with parameters $\mu$ and $\sigma$. We call this a **statistical model**, and our goal is to find the best estimate for one or more of the parameters based on a sample. One way to approach this uses likelihood functions.

Recall that if $X_1, \ldots, X_n$ are independent, identically distributed random variables with common PDF or PMF $f(x)$, then the joint PDF or PMF for the random vector $X = (X_1, \ldots, X_n)^T$ is:

$$f(x_1, x_2, \ldots, x_n) = f(x_1)f(x_2)\cdots f(x_n).$$

If we think of our sample as $n$ independent observations $x_1, \ldots, x_n$ from a population, then we can use the joint density above to get a likelihood function for the unknown parameter(s) in a statistical model. The **maximum likelihood estimate** (MLE) for the parameter(s) is the value of the parameter(s) that maximizes the likelihood function.

1. Suppose that $X_1, X_2$ and $X_3$ represent independent observations all with the Pois($\lambda$) distribution.

   (a) If $x_1 = 4$, $x_2 = 5$, and $x_3 = 3$, then what is the likelihood function $L(\lambda \,|\, x_1, x_2, x_3)$, that is, what is the likelihood function for $\lambda$ given the observed values of $x_1$, $x_2$, and $x_3$?

   (b) What is the maximum likelihood estimate for $\lambda$?

2. Show that the MLE for $\lambda$ based on $n$ independent observations from a Pois($\lambda$) distribution is always the average of $x_1, \ldots, x_n$.

3. Suppose we think that heights of students at one college are normally distributed with unknown mean $\mu$. If we have reason to assume that the standard deviation is $\sigma = 3$ inches, then we can use a sample to find the maximum likelihood estimate for $\mu$. If we find the heights $x_1, \ldots, x_n$ of a sample of $n$ randomly selected students, then show that the sample mean $\bar{x}$ is the MLE for $\mu$.