# Rules of Algebra

*For adding and multiplying real numbers.*

### Addition Rules

1. **Associative** $a + (b + c) = (a + b) + c$
2. **Commutative** $a + b = b + a$
3. $0$ is the **Additive Identity**.
4. **Additive Inverses** are negatives:

$$a + (-a) = 0.$$

### Multiplication Rules

1. **Associative** $a(bc) = (ab)c$
2. **Commutative** $ab = ba$
3. $1$ is the **Multiplicative Identity**.
4. **Multiplicative Inverses** are reciprocals:

$$a\left(\frac{1}{a}\right) = 1.$$

### Distributive Law

$$a(b + c) = ab + ac$$

## Notes

- There are no extra rules for subtraction and division. Subtraction is just addition by negatives. Division is just multiplication by reciprocals.
- You don't need to memorize the names associative and commutative. Just know that if you have several terms added together, then neither the order of the terms nor the order you try to add them matters. Same with several factors multiplied together.
- The distributive law controls everything about how addition and multiplication mix together.

**Terms** are numbers and expressions that are being added/subtracted.

**Factors** are numbers and expressions that are being multiplied/divided.

  - Distribution expands factors into terms.

$$a(b + c) = ab + ac \qquad (x + 2)(x + 3) = x^2 + 5x + 6$$

  - Factoring un-distributes terms back into factors.

$$a(b + c) = ab + ac \qquad (x + 2)(x + 3) = x^2 + 5x + 6$$

  - Factors cancel in fractions.

$$\frac{\cancel{(x-3)}(x+5)}{(x+1)\cancel{(x-3)}} = \frac{(x+5)}{(x+1)} \qquad \text{and} \qquad \frac{\cancel{4}\cancel{x}y}{\cancel{12}x^{\cancel{2}}} = \frac{y}{3x}$$

  - **(Common mistake)** Don't cancel terms in fractions!

$$\underbrace{\frac{4x+3}{4x+7} \neq \frac{3}{7}}_{\text{can't cancel the } 4x \text{ term}} \qquad \text{and} \qquad \underbrace{\frac{4x+3}{4x+7} \neq \frac{x+3}{x+7}}_{\text{can't cancel the 4 either}}$$

# Exponent Rules

**Powers represent repeated multiplication**

$$a^m = \underbrace{a \cdot a \cdot a \cdots a}_{m\text{-copies}}.$$

Therefore these rules are true:

1. $a^0 = 1$
2. $(a^m)(a^n) = a^{m+n}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^n = a^n b^n$
6. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

**Negative Powers**

Negative powers are reciprocals.

$$a^{-n} = \frac{1}{a^n}$$

**Fractional Powers**

Radicals are fractional powers.

$$\sqrt[n]{a} = a^{1/n}$$

## Notes

- Every exponent rule corresponds to a fact about addition & multiplication if you just change the powers to products and the multiplication/division to addition/subtraction. For example:

| | | |
|---|---|---|
| $\underbrace{a^0 = 1}_{\text{1 is the multiplicative identity}}$ | is just like | $\underbrace{0a = 0}_{\text{0 is the additive identity}}$ |
| $\underbrace{(a^m)(a^n) = a^{m+n}}_{(m+n)\text{-copies of } a \text{ multiplied together}}$ | corresponds to | $\underbrace{ma + na = (m+n)a}_{(m+n)\text{-copies of } a \text{ added together}}$ |
| $\underbrace{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}_{n\text{-copies of } \frac{a}{b} \text{ multiplied together}}$ | corresponds to | $\underbrace{n(a-b) = na - nb}_{n\text{-copies of } (a-b) \text{ added together}}$ |

- Simplify rational powers by splitting them up.

$$8^{2/3} = (8^{1/3})^2 = (2)^2 = 4$$

- You can move factors from the top of a fraction to the bottom (or vice versa) by making the power of the factor negative.

$$\frac{x^2(x-5)}{(x+4)^2} = x^2(x-5)(x+4)^{-2} \text{ and also } \frac{x^2(x-5)}{(x+4)^2} = \frac{x^2}{(x-5)^{-1}(x+4)^2}$$

- Powers distribute to factors.

$$(ab)^3 = a^3b^3 \qquad \text{and} \qquad \sqrt{9x^2} = 3x$$

- **(Common mistake)** Powers do not distribute to terms!

$$(a+b)^3 \neq a^3 + b^3 \qquad \text{and} \qquad \sqrt{9 + x^2} \neq 3 + x$$

- **(Common mistake)** Negative powers don't mean the result is negative. And fractional powers don't mean the result is a fraction. Don't confuse powers with multiplication!