

Homework 5 - Math 140

Name: _____

Due by 5:00pm Monday, March 22. Send a PDF with your solutions to blins@hsc.edu.

Calculate the following derivatives.

1. $\frac{d}{dx} 6\sqrt{x}$

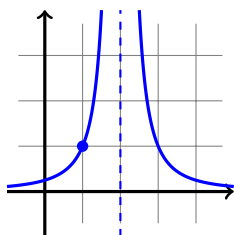
2. $\frac{d}{dx} \frac{1}{3x^2}$

3. $\frac{d}{dt} 16t^2 - 64t + 100$

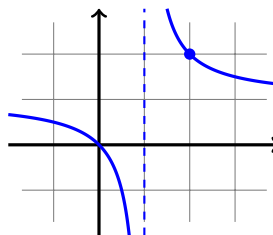
4. $\frac{d}{dp} \left(\frac{1}{p-1} \right) (p^{-2} - p^2)$

Find the slope of the tangent line for the following functions at the indicated point.

5. $y = \frac{1}{x^2 - 4x + 4}$ at $x = 1$.



6. $y = \frac{x}{x-1}$ at $x = 2$.



Where do the following functions have horizontal tangent lines?

7. $f(x) = \frac{x^3 - 3x^2}{2}$

8. $g(x) = (x - 6)(x^2 - 36)$

Solve the following.

9. A company estimates that if they hire L workers, the output of their factor will be $Q(L) = 600L^{2/3}$. Find the derivative $Q'(x)$, then use $Q'(1000)$ to estimate how much output will increase if they have 1,000 workers and hire one more.
10. A ball thrown in the air has a height of $h(t) = 6 + 29t - 16t^2$ feet, where t is time in seconds. If the ball hits the ground at $t = 2$ seconds, calculate how fast the ball is falling then. Recall that velocity is the derivative of position.
11. A fertilizer company estimates that if they produce x tons of fertilizer, their revenue will be $R(x) = \frac{2x}{(1 + \frac{1}{200}x^2)}$ (in thousands of dollars). Find the marginal revenue $R'(10)$. If the company is currently producing 10 tons of fertilizer, should they increase or decrease production to increase revenue?
12. Suppose 6 lbs. of salt is dissolved in a tank containing 50 gallons of water. If the amount of water in the tank is increasing at 1 gallon per minute, then the concentration of the salt is $C(t) = \frac{6}{50 + t}$ pounds per gallon. What is the rate at which the concentration is decreasing after 10 minutes?

In 2021, the Virginia state income tax for individuals is calculated as follows.

Taxable Income	Tax Calculation
0 to \$3,000	2%
\$3,000 to \$5,000	\$60 + 3% of excess over \$3,000
\$5,000 to \$17,000	\$120 + 5% of excess over \$5,000
\$17,000+	\$720 + 5.75% of excess over \$17,000

13. The derivative of $T(x)$ is called the marginal tax rate. Find the formula for $T(x)$ when x is over \$17,000. Then find the derivative $T'(x)$ for that tax bracket.