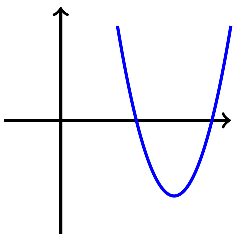
Homework 6 - Math 140 **Name:** ________________________

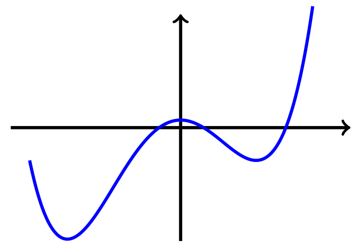
Due by 5:00pm Monday, April 12. Send a PDF with your solutions to `blins@hsc.edu`.

For each of the following functions, find the intervals of increase and decrease. Use the graphs to check your answers.

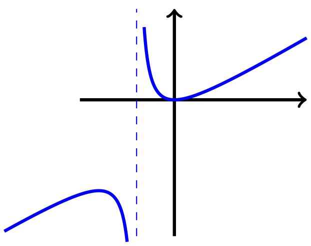
1. $y = x^2 - 6x + 8$



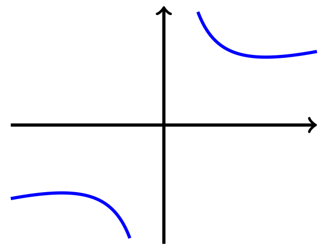
2. $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - 3x^2 + 1$



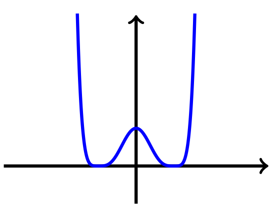
3. $g(x) = \dfrac{x^2}{x+1}$



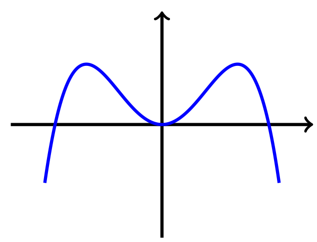
4. $h(x) = \dfrac{x}{3} + \dfrac{3}{x}$



5. $y = (x^2 - 1)^4$



6. $F(x) = x^2(8 - x^2)$



Find the absolute max & min of each function on the given intervals.

7. $y = \dfrac{1}{3}x^3 - 9x + 2$ on $[0, 4]$.

8. $y = x + \dfrac{9}{x}$ on $[1, 10]$.

9. $f(x) = x^2(3 - x)$ on $[0, 4]$.

10. $y = \sqrt{3x^2 - x^3}$ on $[0, 3]$.

11. A ball thrown in the air has a height of $h(t) = 6 + 32t - 16t^2$ feet, where t is time in seconds. How high is the ball at the highest point in its trajectory?

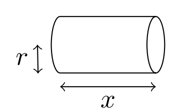
12. The average cost of running a factory is $AC(x) = \frac{600}{x} - 5 + \frac{x}{6}$, where x is the level of production. This function has one critical point at $x = 60$. Use the second derivative test to show that $x = 60$ minimizes the average cost.

13. A potato farmer estimates that they can get $8 per bushel for their potatoes on July 1st. On July 1st, the farmer has 60 bushels of potatoes in the field. For each day the farmer waits to harvest, the price of potatoes will fall by 10 cents per bushel. But if the farmer waits, they can increase their harvest by 1 bushel of potatoes per day. Find a formula for the farmer's revenue as a function of the number of days x they wait to harvest. Hint: *both price and quantity are linear functions of* x.

14. Continuing the last problem, how many days should the farmer wait to harvest their potatoes to maximize revenue?

15. Use the second derivative test on the previous problem to verify that your answer does maximize revenue.

16. According to postal regulations, the length plus the circumference of a cylindrical package sent by 4th class mail cannot exceed 108 inches. Let x be the length, and let r be the radius of the package. So the largest possible cylindrical packages must satisfy this constraint equation: $x + 2\pi r = 108$. Use this constraint to find a formula for the volume of a cylindrical package as function of radius only. (Recall that volume of a cylinder is $V = \pi r^2 x$).



17. Find the radius that maximizes the volume in the previous problem.

18. Find the intervals of increase and decrease for the volume function from the previous problem.