

Homework 8 - Math 140

Name: _____

Due by 5:00pm Friday, April 30. Send a PDF with your solutions to blins@hsc.edu.

Find both partial derivatives of the following functions.

1. $h(x, y) = x^2 + 2xy + y^2$

(a) $\frac{\partial h}{\partial x} =$

(b) $\frac{\partial h}{\partial y} =$

2. $f(x, y) = x^2 e^y$

(a) $\frac{\partial f}{\partial x} =$

(b) $\frac{\partial f}{\partial y} =$

3. $g(x, y) = \frac{y}{x+y}$

(a) $\frac{\partial g}{\partial x} =$

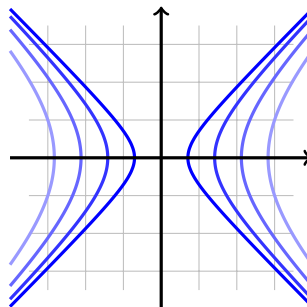
(b) $\frac{\partial g}{\partial y} =$

4. $z = (x^2 + y)^{1/2}$

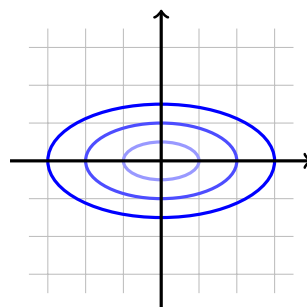
(a) $\frac{\partial z}{\partial x} =$

(b) $\frac{\partial z}{\partial y} =$

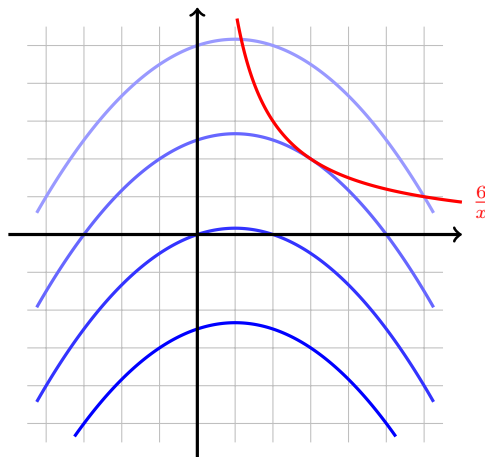
5. Several level curves for the function $f(x, y) = x^2 - y^2$ are shown below. Find the partial derivatives at the point $(1, 1)$ and draw an arrow starting at the point $(1, 1)$ that shows the direction of steepest ascent.



6. Several level curves for the function $f(x, y) = x^2 + 4y^2$ are shown below. Find the partial derivatives at the point $(-2, 1)$ and draw an arrow starting at the point $(-2, 1)$ that shows the direction of steepest ascent.



7. The graph below shows several level curves of the function $f(x, y) = x^2 + 2x - 6y$, and also shows a constraint curve $y = \frac{6}{x}$. Without doing any math, estimate the coordinates of the point on the constraint curve where $f(x, y)$ is maximized. Draw a large dot at that point.



8. Nitric oxide is a gas can be combined with hydrogen to make oxygen and water. The rate of the chemical reaction is $r = kx^2y$ where x is the concentration of nitric oxide and y is the concentration of hydrogen. Find the two partial derivatives r_x and r_y and k is a constant.
9. A cupcake shop can produce $Q(x, y) = 100x^{1/2}y$ dollars worth of cupcakes in a day where x is hours of labor and y is the number of ovens they have running. Find the two partial derivatives Q_x and Q_y when $x = 16$ and $y = 1$.
10. Using the partial derivatives from the previous problem to estimate how much more money the cupcake shop could make if they purchase a second oven and hire a part-time worker who will work 4 extra hours per day. Use the formula:

$$\Delta Q = \frac{\partial Q}{\partial x} \Delta x + \frac{\partial Q}{\partial y} \Delta y.$$

11. A 10-year bond lets you invest x dollars at an interest rate of r . The value of the bond when it matures will be $B(x, r) = x(1 + r)^{10}$. Find the partial derivatives of B with respect to x and r . Then use a calculator to find the values of the two partial derivatives when $x = 100$ and $r = 0.04$.
12. A factory employs two types of workers. They have x skilled workers and y unskilled workers. The total output of the factory is $Q(x, y) = 10x^{0.6}y^{0.4}$. Find the marginal productivity of skilled and of unskilled workers when $x = 50$ and $y = 50$. (Recall that marginal productivity of an input is just the partial derivative of output with respect to that input).