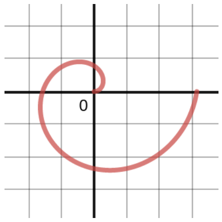
Homework 8 - Math 142 **Name:** ______________________

Due by 5:00pm Friday, April 16. Send a PDF with your solutions to `blins@hsc.edu`.

1. Set up a definite integral that represents the length of Archimedes spiral (shown below), which is given by the parametric equations $x(t) = t\cos t$, $y(t) = t\sin t$ from $t = 0$ to $t = 2\pi$. You don't need to calculate the integral.



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2. It is a fact that $\displaystyle\int \sqrt{1+x^2}\,dx = \tfrac{1}{2}x\sqrt{1+x^2} + \tfrac{1}{2}\ln(x+\sqrt{1+x^2}) + C$. Use this fact to find the length of the spiral in problem 1.

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3. Show that the length of Archimedes spiral is the same as the length of the parabola $y = \frac{1}{2}x^2$ from $x = 0$ to $x = 2\pi$.

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4. Write down the formula for a Riemann sum with 1000 rectangles that estimates the area under the standard normal distribution $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ from $x = 0$ to $x = 1.5$. Then use a computer to find the sum.

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5. Show that the function

$$f(x) = \begin{cases} (x^3 - x^2 + x)e^{-x} & \text{if } x > 0, \\ 0 & \text{otherwise,} \end{cases}$$

is not a valid probability density function because the area under the curve is not 1. Then find the constant c such that $cf(x)$ is a valid PDF.

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6. Find the length of the curve $y = \ln(\cos x)$ from $x = 0$ to $x = \frac{\pi}{4}$.

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7. Consider a random variable with probability density function:

$$f(x) = \begin{cases} \frac{2}{\pi}\left(\frac{1}{1+x^2}\right) & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Show that the expected value (i.e., theoretical average) of this random variable is infinite.

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8. Suppose that a light bulb will last x years where x has the probability density function $f(x) = \frac{1}{2}e^{-x/2}$. Find the probability that the light bulb lasts at least 3 years.
