Math 444 - Antiderivatives   **Name:** _____

1. Integrate the function $f(z) = \frac{1}{z^4}$ on the unit circle (oriented counterclockwise).

2. Does the function $f(z) = \frac{1}{z^4}$ have an antiderivative? What is it, and what is the largest domain in $\mathbb{C}$ where it is the antiderivative?

3. Explain why $\displaystyle\int_{\gamma} 2z \exp(z^2)\, dz = 0$ for any smooth closed path γ in $\mathbb{C}$.

4. Show that $F(z) = \frac{i}{2}\operatorname{Log}(z+i) - \frac{i}{2}\operatorname{Log}(z-i)$ is an antiderivative of $\dfrac{1}{1+z^2}$ on the open right half-plane (the set of $z \in \mathbb{C}$ such that $\operatorname{Re}(z) > 0$).