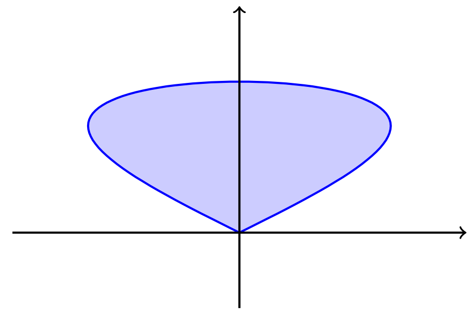
Math 444 - Cauchy's Theorem **Name:** ______________________

1. The standard parametrization of the unit circle is $z = e^{it}$. In this parametrization, what are the differentials dx, dy, and dz?

2. Use Green's theorem to compute $\oint_C y^2\,dx - x^2\,dy$ where C is the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

3. Find a vector field $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ for which $\operatorname{curl} F = 1$ everywhere on $\mathbb{R}^2$. Hint: Let $\mathbf{F}(x,y) = (P(x,y), Q(x,y))$ and choose the simplest functions P and Q you can think of that make the curl equal one everywhere. There is more than one correct answer.

4. Use your answer to the last problem to calculate the area of the region D enclosed by the curve $(\sin 2t, \sin t)$ with $0 \leq t \leq \pi$. Hint: Use Green's theorem to calculate the line integral $\oint_C P dx + Q dy$ in order to find $\iint_D 1\, dA$.



5. Use Green's theorem to prove Cauchy's integral theorem when γ is a simple closed curve, D is the region enclosed by γ, $z = x + iy$, and $f = u + iv$ where both partial derivatives of u and v are defined and continuous in an open set containing D.