Math 444 - Basic Arithmetic in $\mathbb{C}$ **Name:** ______________________

Simplify each of the following expressions as much as you can. Show your work. No calculators.

1. i^{14}
2. $(5i)(-2i)(3i)$
3. $(3+i)^2$
4. $\text{Im}\left(\frac{12}{5-i}\right)$
5. $(3-2i)(4+i)$
6. $\frac{1-i}{1+i}$
7. $\left|\frac{1}{5+12i}\right|$
8. $\overline{(3+4i)(1-i)}$
9. $\overline{e^{i\frac{\pi}{3}}}$

Convert the following from rectangular to polar form.

10. $\frac{1}{2}+\frac{\sqrt{3}}{2}i$
11. $i-1$
12. $\frac{i}{1+i}$

Convert the following from polar to rectangular form.

13. $e^{5\pi i/3}$
14. $(\sqrt{3}\,e^{7\pi i/12})(\sqrt{12}\,e^{29\pi i/12})$

Convert to polar or rectangular form to evaluate the following.

15. $\sqrt{2i}$
16. i^i
17. $\text{Re}\left(2e^{\pi i/6}\right)$
18. $(i-1)^6$
19. $|1-e^{i\frac{\pi}{4}}|$

20. We are going to find the roots of the polynomial equation $z^2+2z+(1-i)=0$ two ways.

    (a) One way to solve the equation is to notice that $z^2+2z+1=(z+1)^2$, so the equation is the same as $(z+1)^2=i$. Solve this by taking the square root of both sides. Remember that all non-zero complex numbers have two square-roots!

(b) Now try using the quadratic formula $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Do you get the same answer as before?

21. **Roots of Unity** An n-th root of unity is a number z such that $z^n = 1$.

    (a) Find all three 3rd roots of unity, and use your answer to factor the polynomial $z^3 - 1$.

    (b) Find a formula for the all n n-th roots of unity.

22. If $z \in \mathbb{C}$ is a root of a polynomial p with real coefficients, then $\overline{z}$ is also a root of that polynomial because $p(\overline{z}) = \overline{p(z)}$. Find an example to show that this is not true for all polynomials with complex coefficients.

23. Prove that the complex-conjugate $\overline{z}$ is the same as the reciprocal $1/z$ if and only if $|z| = 1$. Hint: First show that $|z|^2 = z \cdot \overline{z}$ for every $z \in \mathbb{C}$.

24. For any complex number $w = re^{i\omega}$, find a formula for all n roots of $z^n - w = 0$.

25. Let $z = 1 + \frac{i}{100}$. Find formulas for the real and imaginary parts of z^n (for any integer n) that don't involve any complex numbers.