Math 444 - Complex Derivatives **Name:** ____________________

1. Use the definition of derivative to show that the function $f(z) = \text{Im}(z)$ is not differentiable anywhere.

2. Use the definition of derivative to find the derivative of $f(z) = \frac{1}{z}$.

3. Let $a, b, c, d \in \mathbb{R}$. Calculate $\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$.

4. Compare the answer to the previous problem to what you get when you compute $(a+bi)(c+di)$. What do you notice?

5. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the function $f(x, y) = \begin{bmatrix} x^2 + y^2 \\ 0 \end{bmatrix}$. Find the Jacobian matrix for this function:
$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix}$.

6. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the function $f(x, y) = \begin{bmatrix} x^2 - y^2 \\ 2xy \end{bmatrix}$. Find the Jacobian matrix for this function.

7. Suppose that $z = x + iy$ where $x, y \in \mathbb{R}$. Show that the real and imaginary parts of $|z|^2$ are given by the two entries of the function $f(x, y)$ in problem 5. Show that the real and imaginary parts of z^2 are given by the two entries of the function $f(x, y)$ in problem 6.