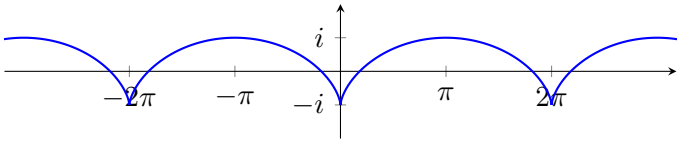
Math 444 - Complex Integrals **Name:** _____

1. Integrate the function $z - \overline{z}$ on the upper half of the unit circle from $z = 1$ to $z = -1$.

2. Integrate the function $\dfrac{1}{z}$ on the vertical line segment from $z = 1$ to $z = 1 - ni$ where n is any positive constant.

3. Find the length of the path $\gamma(t) = t - ie^{-it}$, $0 \leq t \leq 2\pi$. This path is called a cycloid, and it is the path taken by a fixed point on a wheel as it makes one rotation.



4. Show that $\lim_{n\to\infty} \left| \int_\gamma \frac{1}{z}\, dz \right| = 0$ when γ is the horizontal line segment from $1 - ni$ to $-1 - ni$.

Use Sage to calculate the following integrals.

5. $\int_\gamma \frac{1}{z}\, dz$ where $\gamma(t)$ is the arc of the parabola $y = x^2 - 1$ from $x = -1$ to $x = 1$. Hint: You can parametrize γ as $\gamma(t) = t + i(t^2 - 1)$.

6. Some integrals are too hard to evaluate exactly, even for a computer. But you can always use a Riemann sum to estimate the value numerically. Use a Riemann sum with $n = 1000$ steps to estimate $\int_\gamma \sin(z)\, dz$ on the spiral path $\gamma(t) = te^{it}$ for $0 \le t \le 1$. To calculate the Riemann sum, you can either use

$$\sum_{k=1}^{n} f(z_k)\Delta z_k \qquad \text{or} \qquad \sum_{k=1}^{n} f(\gamma(t_k))\gamma'(t_k)\Delta t.$$

Please include your code with your solution for this problem.