Math 444 - Complex Logarithms **Name:** ______________________

1. Let $R = \{z \in \mathbb{C} : \frac{1}{e} < |z| < e\}$. Describe the shape of the image of R after you apply the principal logarithm to R? Draw a sketch of the set you get.

2. If $z = x+iy$ and $\text{Arg}(z) \in [-\pi/2, \pi/2]$, then $\text{Log}(z) = \ln\sqrt{x^2+y^2} + i\arctan(y/x)$. Use this formula to find the Jacobian matrix for $\text{Log}(z)$. Recall that $\ln\sqrt{x^2+y^2} = \frac{1}{2}\ln(x^2+y^2)$ and $\frac{d}{dt}\arctan(t) = \frac{1}{1+t^2}$.

3. For the multiple-valued logarithm, there a difference between the set of all values of $\log(i^2)$ and the set of all values of $2\log i$. Find both sets and explain what it is.

Find all solutions to the following equations.

4. $\text{Log}(z) = i$

5. $\exp(z) = \pi i$

6. $z^5 = e^2$

7. $z^2 = 4 - 4i$

8. $z^n + 1 = 0$

9. Let $a, b \in \mathbb{C}$ with $a \neq 0$. Prove that the set $\exp(b \log a)$ contains only a single value if and only if b is an integer. Hint: Find a formula for every element in the set $\exp(b \log a)$, and then prove that that formula gives only one value if and only if b is an integer.

10. Fix $c \in \mathbb{C}$. Use the definition of z^c (i.e., $z^c := \exp(c \,\text{Log}\, z)$) to find the derivative $\frac{d}{dz} z^c$.