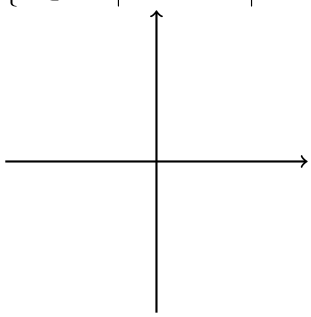
Math 444 - Geometry & Topology in $\mathbb{C}$ **Name:** ______________________

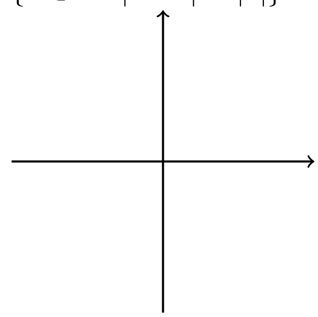
Use the axes provided to sketch each of the following sets in the complex plane. Then circle each topological property that applies to the set.

1. $\{z \in \mathbb{C} : |z - 3 - 4i| = 5\}$



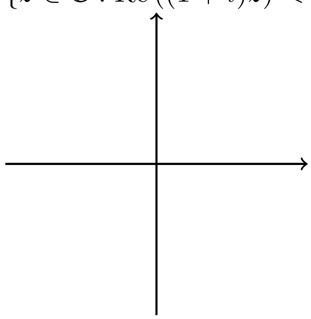
A. Open
B. Closed
C. Bounded
D. Connected

2. $\{z \in \mathbb{C} : |z - i| = |z|\}$



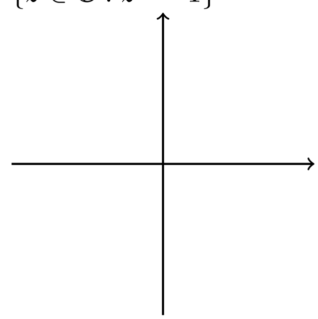
A. Open
B. Closed
C. Bounded
D. Connected

3. $\{z \in \mathbb{C} : \text{Re}\,((1+i)z) < 2\}$



A. Open
B. Closed
C. Bounded
D. Connected

4. $\{z \in \mathbb{C} : z^5 = 1\}$



A. Open
B. Closed
C. Bounded
D. Connected

If $\alpha, \beta \in \mathbb{C}$ and $r > |\alpha - \beta|$, then $\{z \in \mathbb{C} : |z - \alpha| + |z - \beta| = r\}$ is an ellipse with foci at α and β.

5. Assume that both α and β are real numbers. Let $z = x + iy$. Convert the complex equation $|z - \alpha| + |z - \beta| = r$ into an equation for x and y without any imaginary numbers. You should get an equation that a graphing calculator like Desmos can graph.

6. What is the length of the major axis of the ellipse?

7. What is the length of the minor axis?

8. What happens if $r < |\alpha - \beta|$? Does that mean there are no points that satisfy the equation, or do you get a different shape?

Give a parametrization for each of the following curves.

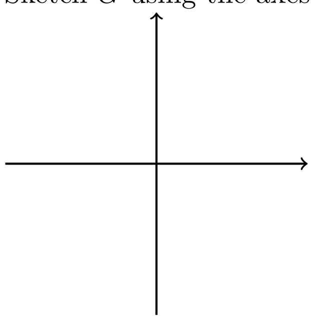
9. The line segment from i to 1.

10. The circle around $z = 2$ with radius 5, oriented clockwise.

11. The triangle with vertices $1 + i$, $i - 1$, and 0, oriented counter clockwise.

Let G be the union of the following two sets, $A = \{z \in \mathbb{C} : 3 < |z| < 5\}$ *and* $B = \{z \in \mathbb{C} : \mathrm{Im}(z) \leq -3\}$.

12. Sketch G using the axes below.



13. What is the boundary of G?

14. What is the interior of G?

15. Is G separated or connected? How can you tell?

16. For any complex numbers $z, w \in \mathbb{C}$, prove that $|wz| = |w|\,|z|$.

17. Use the triangle inequality to prove that if $|w - z| < \delta$, then $|w + z| < \delta + 2|z|$.

18. If $|w - z| < \delta$, show that $|w^2 - z^2| < \delta(\delta + 2|z|)$. Hint: The difference of squares formula works for complex numbers: $w^2 - z^2 = (w - z)(w + z)$.

19. For any $\epsilon > 0$, the set $H = \{z \in \mathbb{C} : |z^2 - 1| < \epsilon\}$ is open. Hint: Use the triangle inequality to show that $|w^2 - 1| \leq |z^2 - 1| + |w^2 - z^2|$. Then show that if $z \in H$, then any w that is close enough to z will also be in H. Can you say exactly how close is 'close enough'?